



Research Article

Role of Darcy Porous Medium on a Radiative Heat Flow Confined to a Heated Porous Microchannel

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Abstract

This study examines the effects of super-hydrophobicity, thermal radiation, and porous media on buoyancy-induced flow of an electrically conducting fluid across a vertical microchannel with alternately heated plates. The theory of simultaneous equations has been employed to determine the closed-form solutions of the stated ordinary differential equations. The computational research revealed that in both cases – Case I (heating the super-hydrophobic surface, or SHS) and Case II (heating the no-slip surface, or NSS) – the heat gradient and fluid motion are significantly enhanced as the values of thermal radiation and Darcy porous parameters increase. However, because of the heat flow channel, increasing the thermal radiation impact decreased the fluid temperature at the NSS. On the other hand, when the magnetic field factor increases, the velocity decreases due to the Lorentz force dragging the molecules, triggering fluid retardation. Our comparison of current research to previously published literature for the limiting scenario reveals a superb relationship that confirms the precision and validity of this work.

Keywords: Thermal radiation, Darcy porous number, Magnetohydrodynamics (MHD), Suction/Injection, Superhydrophobic slip, Slit microchannel

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Introduction

Natural convective flow issues in corrugated tunnels with permeable bedding have been the subject of several engineering research projects. These studies cover a wide range of topics, including solid matrix heat exchangers, oil extraction, biomedical engineering, drying processes, underground disposal of nuclear waste materials, geothermal engineering, building insulation, electronic cooling, manufacturing processes, and solar collector design, to name a few (Choudhary and Ray, 2024). Given this, Hamza *et al.* (2025) examined the effect of a Darcy porous medium on a magnetized, exponentially heated fluid in an alternately heated superhydrophobic microchannel. Daramola *et al.* (2024) highlighted the impact of viscous and Darcy dissipation on the thermal and hydrodynamic behavior of steady natural convection in a fully developed incompressible fluid passing through a vertical composite tube with asymmetric thermal boundary conditions. The study simulates fluid flow for both water and air working fluids using the Brinkman-Darcy extended model. In their investigation of an unsteady MHD natural convection flow in a vertical channel filled with porous materials, Ajibade *et al.* (2023) came to the conclusion that velocity profiles grow as permeability parameters increase. In a saturated porous media, Mahdy *et al.* (2021) examined the convective flow of a micropolar hybrid nanofluid via a vertical radiating permeable plate. They found that the skin friction coefficient and Nusselt number are improved by increases in micropolarity and nanoparticle volume percentage. In a permeable enclosure with two cylinders embedded in a hollow, Barnoon *et al.* (2019) investigated heat transfer and non-Newtonian nanofluid flow both with and without thermal radiation. They discovered that depending on the cylinder's size, heat transport might either increase or decrease. Venkatadri *et al.* (2023) reported that free convective flows in a right-angled trapezoidal cavity filled with Cu-water nanofluid material and saturated with a porous bed showed that the flow rate disperses over the porous region as the Darcy number (Da) rises. Hamza *et al.* (2022) investigated the effects of magnetohydrodynamics on the unsteady fluid flow of an exothermic reaction in a vertical channel filled with porous material. Their research discovered that increasing permeability improved fluid flow, resulting in a rise in fluid velocity. Furthermore, when the Biot number grows, so do the channel's surface and interior temperatures. According to Girish and Sankar (2020), the baffle location, Darcy number, modified Grashof number, and Brinkman number all have a substantial impact on flow. However, the fall in the heat transfer coefficient is less substantial as the thickness of the porous medium decreases.

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Due to its importance in building various sophisticated energy conversion systems that can function at high temperatures, the idea of the thermal radiation effect—the electromagnetic wave radiation that a surface produces due to its heat—is receiving more and more attention, particularly when a magnetic field is applied (Jamaludin *et al.* 2020). Other real-world uses include solar energy, nuclear power plants, spaceship aerodynamics, and more. Several academics have investigated the effects of heat radiation in various physical configurations to achieve this goal. Shah *et al.* (2023) discussed the impact of heat-dependent thermodynamic fluid characteristics in MHD Casson flow influenced by the coexistence of thermal and chemical processes, keeping in mind the many benefits of thermal radiation. The hydromagnetic natural flow of an electrically conductive Casson fluid caused by the action of heat radiation in an upright porous channel was suggested by Ojmeri *et al.* (2023). Goud *et al.* (2023) examined the analysis of transient MHD flow through a permeable medium across an upright plate in the context of the coexistence of viscous dissipation and thermal radiation effects, while Gireesha *et al.* (2020) described the effects of thermal radiation and free convection on the flow of a water-based hybrid nanofluid containing nanoparticles through a porous vertical channel using Darcy's model. Hasan *et al.* (2020) used the approach of an indeterminate coefficient to assess the effects of thermal radiation, a heat source, and an induced magnetic field on the natural convective flow of a pair stress fluid in a flux-isothermal vertical channel. Bejawada and Nandeppanavar (2022) investigated the impact of the MHD heat transfer problem on the micropolar fluid through a vertically permeable moving plate in the presence of thermal radiation. A theoretical study of radiative-convection effects on MHD fluid flow in a heated square enclosure with a non-Darcy square cavity was described by Parthiban and Pasad (2023).

The engineering sciences and technology sectors are paying close attention to evaluating the behavior of the special interactions when magnetized convection travels in a superhydrophobic surface (SHS) microchannel. Because of the enormous slip brought on by liquid/solid connections, superhydrophobic (SHO) walls can reduce frictional force in the flow, which makes them a useful condition to assess the amount of drag force induced by the slip length (Jha and Gwandu 2020). Numerous systematic buildup investigations have examined the impact of MHD convective fluxes across a heated SHO microchannel. The significance of coupled hydromagnetic flow with nonlinear variables through an upright microchannel covered with superperhydrophobic characteristics was emphasized by Ojmeri and Ahmad (2025) in light of these worries. Hatte and Pitchumani (2021) precisely examined the effects of heat transfer through a cylinder with a non-wetting surface using fractional rough wall characteristics. They concluded that, contrary to common belief, hydrophobic coatings can provide exceptional thermal efficiency in some fluid flow scenarios, and that super-hydrophobicity—which is characterized by the greatest contact angles—rarely leads to peak convective heat transfer behavior. An analytical study of a chemically reactive hydromagnetic fluid impacted by heat production and absorption passing through a superhydrophobic microchannel that is heated

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alternately was recently reported by Ojmeri *et al.* (2024). Ojmeri and Onwubuya (2023a, 2023b) conducted a thorough investigation of the performance of superhydrophobic slip and temperature jump circumstances in a slit microchannel influenced by Arrhenius kinetics, mixed convection, and porous medium. They inferred from their data that the super-hydrophobicity effect increases fluid velocity in the microchannel. Jha and Gwandu (2018) performed theoretical research on MHD natural convection in a vertical slit microchannel exhibiting a super-hydrophobic slip effect and temperature spike. Their findings revealed that when there is a temperature jump and no super-hydrophobic slip, or both, the maximum upward velocity obtained by heating the super-hydrophobic wall is smaller than that obtained by heating the no-slip surface. The maximum velocities are comparable when neither is present. Jha and Gwandu (2019) then used non-linear Boussinesq approximation methods to study the free convection flow of an electrically conducting fluid in a vertical slit microchannel that was impacted by super-hydrophobic slip and temperature jumps. According to the computational results, increasing the temperature jump coefficient causes a decrease in temperature when the super-hydrophobic surface is heated and an increase in temperature when the no-slip surface is heated. Yale *et al.* (2023) used the regular perturbation approach to investigate the effect of viscous dissipative fluid on hydromagnetic natural convection in a heated superhydrophobic microchannel.

According to the reviewed literature, insufficient attention has been paid to the analysis of porous media with thermal radiation effects via an alternatively heated vertical superhydrophobic microchannel, which has constrained prior research. To fill the knowledge gap in the literature, the main goal of this article is to expand on the work of Hamza *et al.* (2023) by conducting a theoretical investigation of a buoyancy-induced flow influenced by thermal radiation and permeability effects across a slit microchannel that is alternately heated and equipped with superhydrophobic slip and temperature jump conditions. The modeled dimensionless governing equations are a collection of linked ordinary differential equations that have been analytically solved using the theory of simultaneous equations. Illustrative graphs have also been sketched to demonstrate the impacts of embedded parameters dictating the flow configuration. The outcomes of this study can find applicability in areas such as cooling of nuclear reactors, solar power collectors, and energy-efficient drying processes. Moreover, many processes in engineering areas occur at high temperatures, and a knowledge of radiation heat transfer becomes very critical for designing pertinent devices. The various propulsion devices for aircraft, missiles, satellites, and space vehicles are examples of engineering areas where this research can find relevance. Additionally, this study can provide wider applications in mechanical microfluidics, micro-electro-mechanical systems (MEMS), microelectronics, and biofluidics than the previous related studies available in the literature. Because of the massive slip obtained from liquid/solid interfaces, SHS can reduce drag in a flow, making it a very useful parameter for measuring the extent of drag reduction based on slip length in oil and gas

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companies, semiconductor manufacturing facilities, and companies that assemble small equipment.

Mathematical Structure of the flow Problem

Consider an electrically conducting fluid traveling slowly upward via a vertical parallel plate micro-channel by convection, with one of the plates heated at a time. One of the surfaces is exceedingly difficult to wet (super-hydrophobic) as a consequence of a unique micro-engineering treatment. The opposing side (the no-slip surface) was not tampered with. Figure 1 depicts the super-hydrophobic surface at $y_0 = 0$ and the no-slip surface at $y_0 = L$. Because the primary focus is on the superhydrophobicity of a surface rather than the flow characteristics, different temperature jump and slip conditions were applied for each plate. A magnetic field with homogeneous intensity $(0, B_0, 0)$ is supposed to be applied in the direction perpendicular to the flow. One of the surfaces is exceedingly difficult to wet (super-hydrophobic) as a consequence of a unique micro-engineering treatment. The opposing side (the no-slip surface) was not tampered with.

The following assumptions were considered under the present work:

- i. The fluid is viscous, incompressible, and electrically conductive.
- ii. The flow is laminar and fully developed.
- iii. A magnetic field of uniform strength B_0 is assumed to be applied in the direction perpendicular to the direction of flow.
- iv. It is assumed that one of the surfaces is extremely difficult to wet (super-hydrophobic)
- v. It is assumed that the flow is considered in terms of thermal radiation and porous medium effects.

The component u is the fluid vertical velocity, and V_0 is the suction/injection velocity. Heat flow along the channel was neglected. Following Hamza *et al.* (2023), under the Boussinesq buoyancy approximation and assuming that the fluid is viscous, electrically conducting, and in the presence of thermal radiation and suction/injection, the controlling equations for the present problems in dimensional form can be as follows:

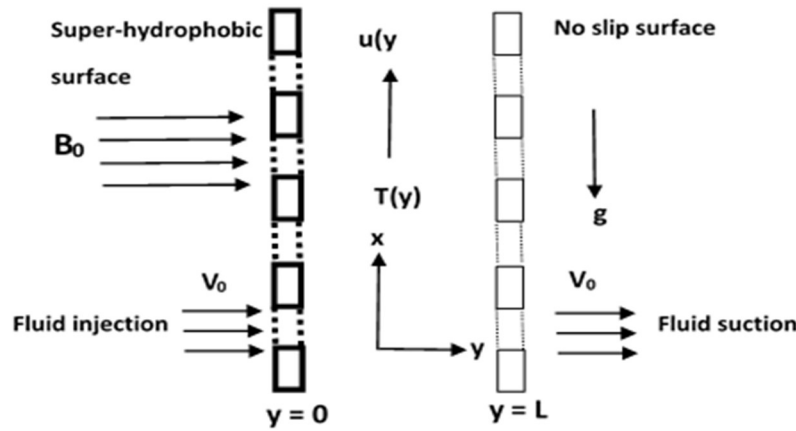


Figure 1: Physical Coordinate of the Flow System

$$V_0 \frac{du'}{dy'} = \nu \frac{d^2 u'}{dy'^2} + g\beta(T' - T_0) - \frac{\sigma B_0^2 u'}{\rho} - \frac{u'}{K} \quad (1)$$

$$V_0 \frac{dT'}{dy'} = \frac{k}{\rho c_p} \frac{d^2 T'}{dy'^2} - \frac{1}{\rho c_p} \frac{dq_r}{dy'} \quad (2)$$

We assume that the appropriate boundary conditions at the walls are:

$$\left. \begin{aligned} u(y') &= \lambda' \frac{du'}{dy'} \\ T(y') &= T_0 + A(T_h - T_0) + \gamma' \frac{dT'}{dy'} \end{aligned} \right\} \quad \text{at} \quad y' = 0$$

$$\left. \begin{aligned} u(y') &= 0 \\ T(y') &= T_0 + B(T_h - T_0) \end{aligned} \right\} \quad \text{at} \quad y' = L \quad (3)$$

Applying the Rosseland approximation, the radiative heat flux qr is indicated by:

$$qr = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \quad (4)$$

Where σ^* and k^* are the Stefan Boltzmann constant and Rosseland mean absorption parameters, respectively. Consider that the temperature difference inside the flow is sufficiently negligible so that T^4 may be considered as a linear function of temperature in a Taylor series about T_0 and ignoring higher terms one can get

$$T^4 \cong 4T_0^3 T - 3T_0^4 \quad (5)$$

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Using equations (4) and (5) equation (6) becomes

$$\frac{k}{\rho c_p} \left(1 + \frac{16\sigma^* T^3}{3kk^*} \right) \frac{\partial^2 T}{\partial y^2} - S \frac{\partial T}{\partial y} = 0 \quad (6)$$

Where $R = 16\sigma^* T^3 / 3kk^*$ is taken as the radiation parameter, so the above equation becomes

$$\frac{k}{\rho c_p} (1 + R) \frac{\partial^2 T}{\partial y^2} - S \frac{\partial T}{\partial y} = 0 \quad (7)$$

And where the dimensionless parameters used are:

$$y = \frac{y'}{L}, u = \frac{u'}{u_0}, \theta = \frac{T - T_0}{T_1 - T_0}, x = \frac{x'v}{UL^2}, (Y, \gamma, \Gamma) = \frac{Y', \gamma', \Gamma'}{L}, M^2 = \frac{\sigma \beta_0^2 L^2}{\rho v} \\ S = \frac{v_0 L}{v}, Pr = \frac{\rho c_p v}{k}, K = \frac{k}{L^2} \quad (8)$$

Incorporating equation (8) in equation (1), (2) and (3), the governing equations now becomes:

$$\frac{d^2 u}{dy^2} - S \frac{du}{dy} - \left(M^2 + \frac{1}{K} \right) u + \theta = 0 \quad (9)$$

$$(1 + R) \frac{d^2 \theta}{dy^2} - S Pr \frac{d\theta}{dy} = 0 \quad (10)$$

The boundary conditions in dimensionless forms are:

$$\left. \begin{aligned} \theta(0) &= A + \gamma \frac{d\theta}{dy}, \quad u(0) = \lambda \frac{du}{dy} \\ \theta(1) &= B, \quad u(1) = 0 \end{aligned} \right\} \quad (11)$$

Where R is the thermal radiation, K is the Darcy porous number, M is the magnetic field intensity, S is the suction/injection parameter, γ is the temperature jump coefficient and λ is the velocity slip condition.

Solution Procedure

The governing equations are a set of ordinary differential equations with constant coefficients. The theory of simultaneous ordinary differential equations is used to address the closed-form solution of the linear ordinary differential equation system. The steady-state temperature and velocity have been obtained as follows:

$$\theta(y) = D_1 + D_2 e^{m_2 y} \quad (12)$$

$$U(y) = E_1 e^{m_3 y} + E_2 e^{-m_4 y} + E_3 + E_4 e^{m_2 y} \quad (13)$$

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The rate of heat transfer and the drag force coefficients are computed as follows:

$$Nu_0 = \frac{d\theta}{dy} = -m_2 D_2 \quad (14)$$

$$Nu_{u_1} = \frac{d\theta}{dy} = -m_2 D_2 e^{-m_2} \quad (15)$$

$$\tau_0 = \frac{du}{dy} = m_3 E_1 - m_4 E_2 - m_2 E_4 \quad (16)$$

$$\tau_1 = \frac{du}{dy} = m_3 E_1 e^{m_3} - m_4 E_2 e^{-m_4} + m_2 E_4 e^{-m_2} \quad (17)$$

Where the constants D_1, D_2, E_1, E_2, E_3 , and E_4 are indicated in the appendix.

Discussion of the Findings

The steady-state study of MHD free convection flow of an incompressible, electrically conducting fluid traveling upward along an isothermally heated porous microchannel, with one surface coated with a super-hydrophobic slip condition and temperature jump, and the other with no slip. The analytical expressions for energy, momentum, heat transfer coefficient, and skin friction have been determined. Several graphs were created to depict and explain the effects of the primary regulating factors on fluid flow and thermal profile for cases I (SHS), heated, and II (NSS), heated. The default values chosen for this present investigation are $R=1$, $\lambda = \gamma = 1$, $S=M=0.5$, and $K=0.1$. To verify the accuracy and correctness of the current investigation, a graphical representation of Hamza *et al.* (2023) in conjunction with the present work is plotted in Figure 2 for the limiting cases where $R = 0$ and $K = 1000$, with the other parameters held at constant values. The comparison revealed an excellent agreement.

With the aid of Figure 3, we comprehend the behavior of fluid velocity as the Darcy porous medium is varied. It is apparent from this figure that the fluid velocity accelerates as the porosity parameter is increased. We observe that, with the growth in permeability of the porous medium, the drag force weakens; because of this, the velocity gradient of the fluid improves. Moreover, this makes sense because when a lot of fluid is moved, more viscous energy is produced. This causes the fluid boundary wall and thickness to grow, which speeds up the movement of the fluid. The deviation of thermal radiation parameter on the dimensionless temperature distribution for fixed value of ($\lambda = \gamma = 1$, $S=M=0.5$ and $R=1$) is displayed in Figure 4. It clearly reveals that, from these figures, the temperature of the fluid is scaled up with an increase in the thermal radiation in case I, while it reduces it in case II. This was due to the trapped warm air packets at the SHS and the direction of the heat flow. With the help of Figure 5, the action of

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thermal radiation on the velocity gradient is described. It can be seen that the thermal radiation effect favors the fluid flow in both cases.

Figure 6 indicates the impact of MHD on the velocity distribution. The graph shows a reduction in the upward velocity (especially the peak velocity) with an increase in magnetic number (when both λ and γ are equal to 1). In other words, it was evident that the magnetic parameter impedes the fluid motion by attracting the molecules, consequently, leading to its deterioration. The functions of thermal radiation on the rate of heat transfer are plotted in Figures 7a and 7b. In Figure 7a, the heat transfer rate is observed to demonstrate a decaying tendency at the SHS surface, while a reverse case is noticed at the no-slip surface ($y=1$) as shown in Figure 7b. Figure 8 plots the implication of thermal radiation on the skin friction. It was plain that rising values of the thermal radiation parameter are seen to encourage the shear stress at SHS, whereas a counter attribute occurs at the NSS.

The response of varying Darcy porous numbers to the frictional force at both plates is plotted in Figure 9. It is noticeable that raising the level of porous parameter encourages skin friction at the plate ($y = 0$), as elucidated in Fig. 10a, whereas in Figure 9b, a counter attribute happens at the plate ($y = 1$). The consequence of fluctuating magnetic number on the skin friction is portrayed in Figure 10. The action of MHD is to strengthen the shear stress at the SHS ($y = 0$), while the opposite phenomenon happens at the NSS ($y=1$).

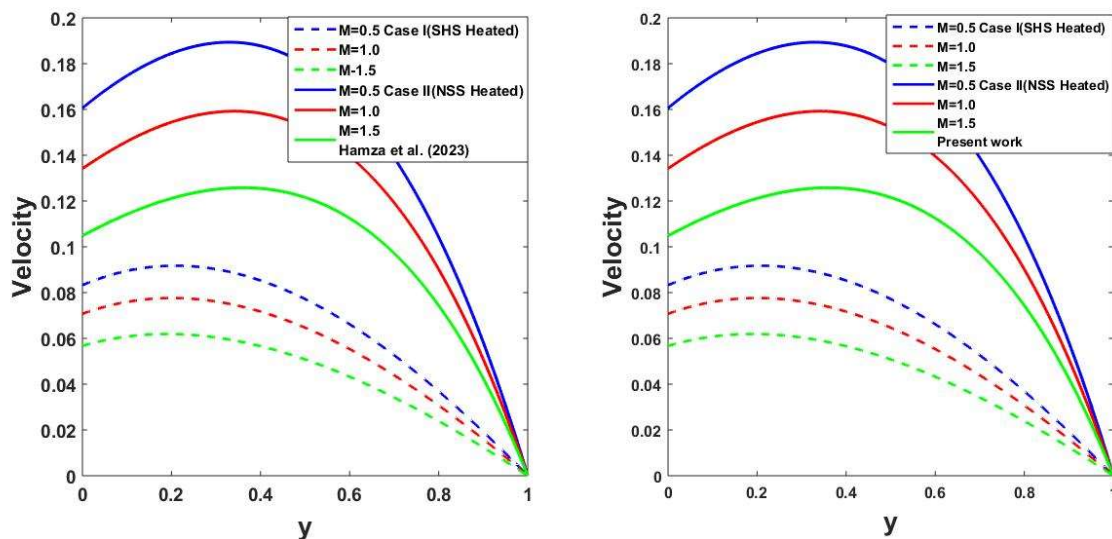


Figure 2 Comparison of Hamza *et al.* (2023) with the current work for constant values of $\lambda=\gamma=1$, $S=0.6$, $Pr=0.025$ when $R=0$ and $K=1000$ for Case I and Case II, respectively

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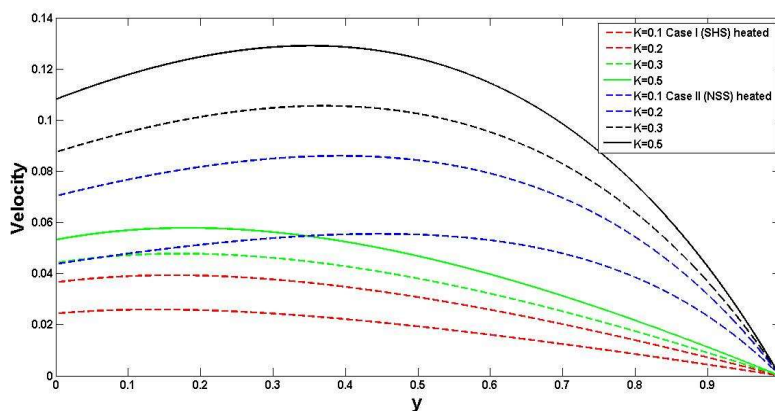


Figure 3: Action of porosity effect on velocity profile

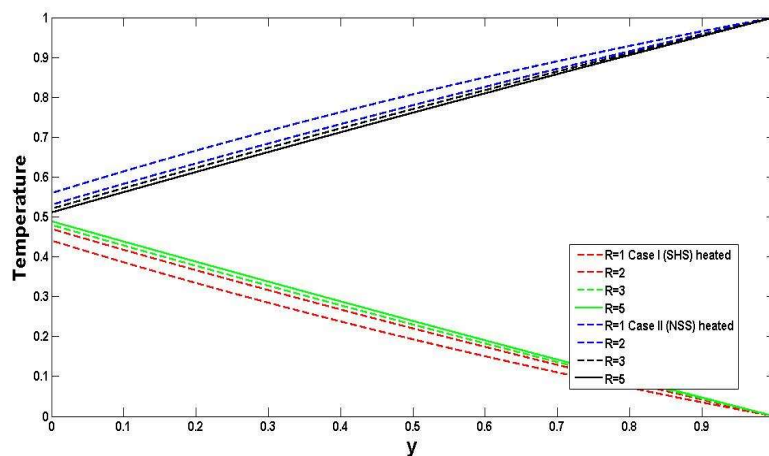


Figure 4: Action of thermal radiation on temperature profile

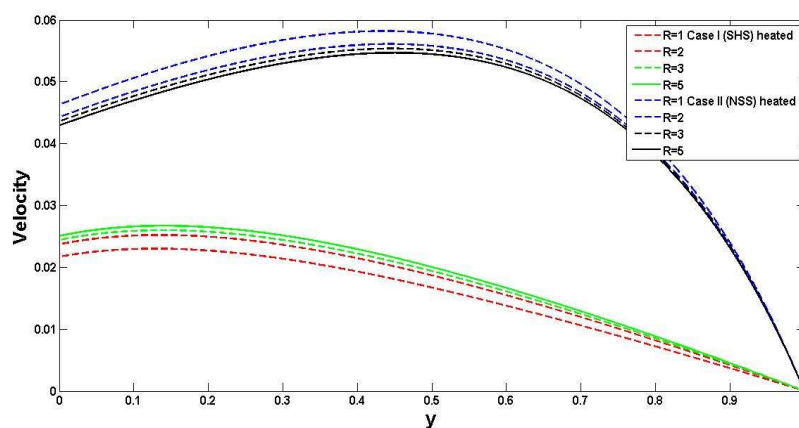


Figure 5 Action of thermal radiation on velocity profile

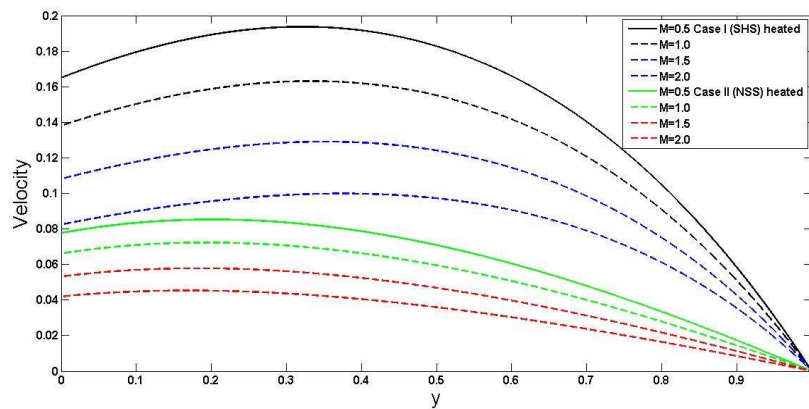


Figure 6: Action of magnetic field effect on velocity profile

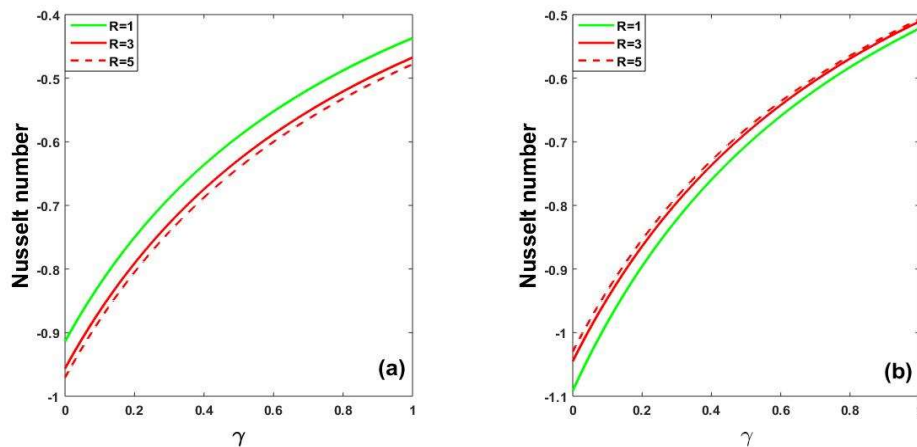


Figure 7: Action of thermal radiation on Nusselt number for (a) lower and (b) upper plates

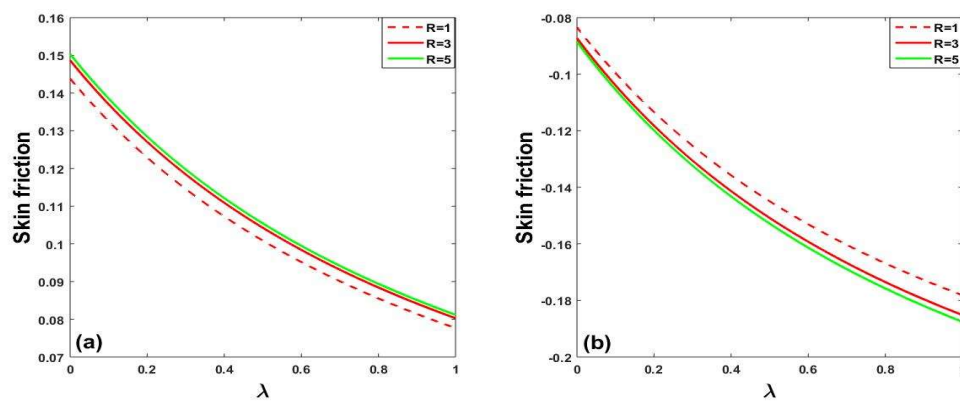


Figure 8: Action of thermal radiation on Skin friction for (a) lower and (b) upper plates

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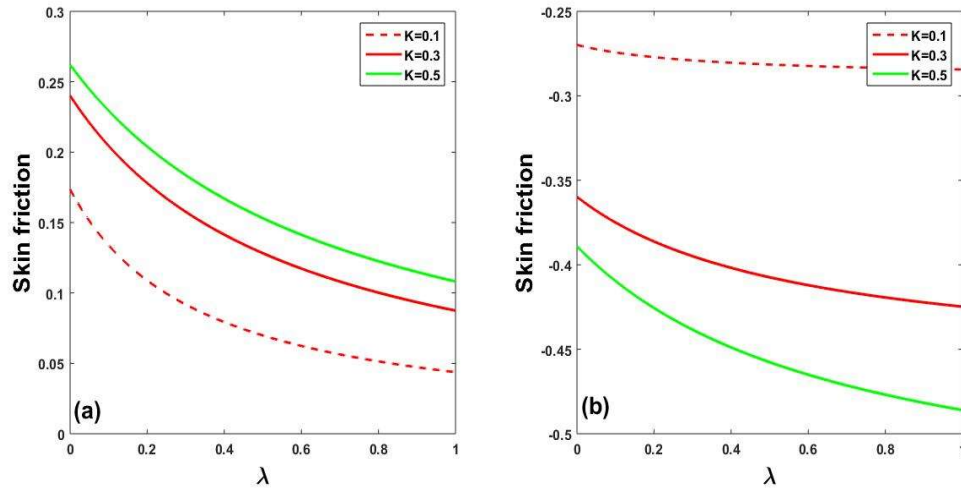


Figure 9: Action of porosity effect on Skin friction for (a) lower and (b) upper plates

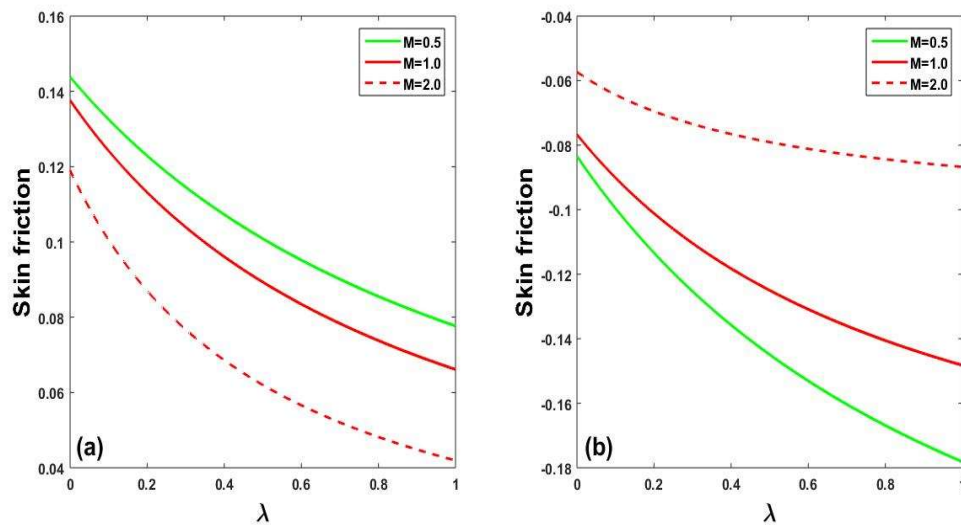


Figure 10: Action of the magnetic field effect on Skin friction for (a) lower and (b) upper plates

Conclusion

This study investigated the combined effects of thermal radiation and permeability factors on a steady, fully developed hydromagnetic flow of an electrically conducting fluid moving vertically through an isothermally heated porous plate microchannel, with one plate exhibiting superhydrophobic slip and temperature jump and the other not. Because the governing equations are a system of ordinary differential equations with constant coefficients, the theory

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of simultaneous ordinary differential equations provided a closed-form solution. The effects of adjusting factors such as porous material, thermal radiation, suction/injection, and magnetic field intensity on fluid flow, temperature, drag force, and heat transfer quantity were given and explored in detail, using illustrated charts. The overview of the findings from this study is given below:

- (i) The fluid temperature is noticeably increased for increasing values of thermal radiation in case I, while the reverse happens in case II. This is attributable to the direction of the heat flow.
- (ii) Thermal radiation and porous medium effects are envisaged to promote the fluid velocity in both cases.
- (iii) The velocity of the fluid is lowered with an increase in the magnetic field number, while the skin friction depicts a diminishing tendency when the SHS is heated; a contrasting behavior was recorded when the NSS is heated.
- (iv) The rate of heat transfer is minimized at the SHS in favor of the thermal radiation parameter, while a contrary behavior is recorded at the NSS. In contrast, the respective counter-attribute is portrayed in sheer stress.
- (v) It was concluded that raising the level of the porous parameter encourages skin friction at the plate ($y = 0$), whereas a counter attribute happens at the plate ($y = 1$).
- (vi) In the future, the impact of mixed convection will be studied on the present model.

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Appendix

$$m_2 = \frac{SPr}{1+R}, \quad m_3 = \frac{1}{2}(S + \sqrt{S^2 + 4(M^2 + 1/K)}), \quad m_4 = \frac{1}{2}(S - \sqrt{S^2 + 4(M^2 + 1/K)}),$$

$$D_1 = \frac{1}{(1 - e^{m_1} - \gamma m_1)}, \quad D_2 = -D_1 e^{-m_2}, \quad E_3 = \frac{-D_1}{(m_2^2 - S m_2 - M^2)}, \quad E_4 = \frac{D_2}{M^2},$$

$$E_1 = \frac{-E_2 a_3 - a_1}{a_2}, \quad E_2 = \frac{a_6}{a_5}, \quad a_1 = E_3 + E_4 - \lambda E_3 m_2, \quad a_2 = 1 - \lambda m_3, \quad a_3 = 1 + \lambda m_4,$$

$$a_4 = E_3 e^{-m_2} + E_4, \quad a_5 = a_2 e^{-m_4} - a_3 e^{m_3}, \quad a_6 = a_1 e^{m_3} - a_2 a_4$$

List of Symbols

B_0 = Constant magnetic flux density [kg/s².m²]
 g = Gravitational acceleration [m/s²]
 h = Channel width [m]
 $C_p C_v$ = Specific heats at constant pressure and constant volume [Jkg⁻¹K⁻¹]
 λ = Dimensionless slip length parameter
 γ = Dimensionless temperature jump parameter
 R = Thermal radiation parameter

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K = Darcy Porous Number

M = Magnetic field

S =Suction/injection parameter

Nu = Dimensionless heat transfer rate

T = Dimensionless temperature of the fluid [K]

T_0 =Reference temperature [K]

T_{∞}^* = The temperature far away from the plate K

u =Dimensionless velocity of the fluid [ms^{-1}]

y = Dimensionless distance between plates

U_0 = Reference velocity [ms^{-1}]

K^* = The mean absorption coefficient [m^{-1}]

β = Thermal expansion coefficient [K^{-1}]

μ = Variable fluid viscosity [$kgm^{-1}s^{-1}$]

k = Thermal conductivity [$m.kg/s^3.K$]

q_r^* = The radiative heat flux [Wm^{-2}]

α = Thermal diffusivity [m^2s^{-1}]

σ = Electrical conductivity of the fluid [s^3m^2/kg]

σ^* = The Stefan-Boltzmann constant [$W m^{-2} K^{-4}$]

ρ = Density of the fluid [kgm^{-3}]

ν = Fluid kinematic viscosity [m^2s^{-1}]