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Research Article

# Effects of Soret and Radial Magnetic Field of a Free Convective Slip Flow in a Viscous Reactive Fluid Towards a Vertical-Porous-Cylinder

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## Abstract

The steady state analysis of a free convection flow of an incompressible viscous reactive fluid through a vertical porous cylinder under the influence of slip flow and Soret effects in the presence of radial magnetic field is performed. Using the perturbation series method, the results for the dimensionless nonlinear differential equations are solved for momentum, temperature, concentration, skin friction, Nusselt number as well as Sherwood Number. The effects of various parameters such as Frank Kamenetskii parameter (λ), Navier slip condition (P), Soret effect (Sr), thermal Grashof number (Gr), solutal Grashof number (Gc) and Hartmann number (M) are presented graphically and discussed in detail. It is observed that, the momentum is enhanced significantly owing to an increase in the values of Frank Kamenetskii, Slip, Soret, thermal Grashof number, solutal Grashof number parameters respectively while a decrease in fluid flow is noticed for increasing values of magnetic field and Prandtl number.

Keywords: Free convection, Navier slip flow, viscous reactive fluids, Soret effect, radial magnetic field, Vertical porous cylinder.

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## Nomenclatures

| | | |
|---|---|---|
| $\beta_0$ | = | Radial magnetic field strength |
| $b$ | = | radius of the cylinder |
| ε | = | Activation energy |
| g | = | Acceleration of gravity |
| Nu | = | Nusselt Number |
| Pr | = | Prandlt number |
| Sc | = | Schmidt number |
| Sr | = | Soret number |
| $C$ | = | Dimensionless Concentration |
| Gr | = | Thermal Grashof number |
| Gc | = | solutal Grashof number |
| s | = | Suction/injection parameter |
| $K_T$ | = | Thermal diffusion ratio |
| μ | = | Viscosity parameter |
| $T_0$ | = | Initial temperature of the fluid |
| r | = | Dimensionless radial coordinate |
| $r_0$ | = | characteristics radius |
| u | = | Dimensionless velocity of the fluid |
| $\nu_0$ | = | Sunction/injection velocity of porous plate |
| P | = | Navier slip parameter |

## Greek Letters

| | | |
|---|---|---|
| $\sigma$ | = | Electrical conductivity |
| ρ | = | Density of the fluid |
| α | = | Thermal diffusivity |
| θ | = | Dimensionless temperature |
| λ | = | Frank Kamenetskii parameter |
| $\mu_e$ | = | Magnetic permeability |
| v | = | Kinematic viscosity of the fluid |
| τ | = | skin-friction |


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## Introduction

Recent interest in the analysis of slip flow phenomenon of a free convective viscous reactive fluid has been mainly motivated by its importance in vertical channels and in modern science and engineering (Sami *et al*. 2015). According to Marek (2004), simulations that involve low-viscosity fluid sand large length scales are the applications of slip boundary condition at the entire wetted surface. Some research works has been done on free convection slip flow in a vertical channel with different thermal conditions and geometries. In view of this, Ahmad *et al*. (2017) presented free convection flow with diffusion-thermo and Navier slip condition in the presence of Newtonian heating in a vertical porous channel. They reported that, diffusion thermo, Navier slip and Newtonian heating substantially influence the fluid flow. Hamza (2016), analyzed transient/steady natural convection flow of an exothermic fluid in a vertical channel under wall Navier slip condition and Newtonian heating using the unconditionally stable and convergent implicit finite difference scheme. The results of his findings revealed that, Navier slip condition, Newtonian heating and Frank-Kamenetskii parameter strongly influence the flow pattern. Sharma (2005) outlined the effect of periodic heat and mass transfer on the unsteady free convection flow past vertical flat plates in the slip flow regime when suction velocity oscillates with time. Kumar (2013), analyzed the magnetohydrodynamic (MHD) boundary layer flow of heat and mass transfer over a stretching sheet with slip effect. Hamza *et al.* (2011) presented unsteady heat transfer of MHD oscillating flow through a porous medium under slip condition. Egunjobi and Makinde (2012), investigated the combined effect of buoyancy force and Navier slip on the entropy generation rate in a vertical porous channel with wall suction/injection. Sedeek and Abdelguid (2004) studied the effect of slip condition on magneto-micropolar fluid with combined forced and free convection in boundary layer flow over a horizontal plate. The combined effects of thermal radiation and MHD of a free convection flow through a moving/stationary vertical plates under the influence of Newtonian heating filled with porous material was analyzed by Ojemeri *et al.* (2018). Osalusi *et al.* (2008) presented the thermo-diffusion and diffusion-thermo effects on combined heat and mass transfer of a steady hydromagnetic convective and slip flow due to a rotating disk in the presence of viscous dissipation and Ohmic heating. Pal and Talukdar (2010) investigated analytically the combined effect of heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction. The combined effects of Navier slip and Newtonian heating on MHD unsteady flow and heat transfer toward a flat plate were presented by Makinde (2012). The influences of buoyancy force and Navier slip on hydromagnetic flow of a Nano fluid over a convectively heated vertical porous plate have been investigated by

Mutuku-Njane and Makinde (2013). They found that, the thermal boundary layer is enhanced by increasing the intensity of Newtonian heating while the cooling effect on the convectively heated plate surface is enhanced by increasing the velocity slip and suction parameter. Similarly, Adesanya (2015) carried out analytical computations of the effect of velocity slip and temperature jump on free convective flow of heat generating/absorbing fluid through a porous vertical channel.

The diffusion of mass due to temperature gradient is called Soret or thermo-diffusion effect and this effect might become significant when large density differences exist in the flow region. For example, Soret effect can be significant when species are introduced at a surface in fluid domain, with a density lower than the surrounding fluid. The Soret effect has been utilized for isotope separation and in a mixture between gases with very light molecular weight and of medium molecular weight. Ahmad *et al*. (2017) investigated steady state mixed convective heat and mass transfer flow of reactive viscous fluid in a vertical porous pipe in the presence of Soret effect. Their results revealed that mixed convection and Soret effect significantly influence the fluid flow in the pipe. The influence of Soret effect on the unsteady motion of MHD mixed-convection flow of a viscoelastic fluid over an infinite vertical plate in the presence of a heat source was investigated by Jha *et al.* (2014). Ahmed (2012) made an exact analysis of the thermal-diffusion effect on combined heat and mass transfer Hartmann flow through a channel bounded by two infinite horizontal isothermal parallel plates. The Soret-driven, free-convection heat-and-mass-transfer flow of a non-Newtonian liquid past a vertical plate in a thermally stratified porous medium was studied by Narayana *et al*. (2013). MHD heat-and-mass-transfer flow over a vertical plate in a porous medium under the influence of Dufour and Soret effects was discussed by Vedavathi *et al*. (2015). Sharma et al. (2014) studied the Soret and Dufour effects on unsteady MHD mixed-convection flow with heat source and Ohmic dissipation.

In recent years, researchers have shown their interest in this emerging field of MHD due to its wide range of applications in astrophysics, geophysics and engineering. These types of studies are important in nuclear power plants, drilling operations, geothermal power generation and space vehicles technology etc. (Vanita and Kumar, 2016). The MHD flow problem in an annulus was first discussed by Globe (1959) who considered fully developed laminar MHD flow in an annular channel. Mixed convection in vertical annulus under a radial magnetic field was considered by Mozayyeni and Rahimi (2012) and it was found that velocity and temperature of fluid can be suppressed more effectively by using external magnetic field. Reddy and Reddy (2009) investigated the study of radiation and mass transfer effects on unsteady MHD free convection flow of an incompressible viscous fluid past a moving vertical cylinder. They deducted the behavior of the velocity, temperature, concentration, skin-friction, Nusselt number and Sherwood number with variations in the governing thermophysical and hydro-dynamical parameters. Ali *et al.* (2013) presented the flow profiles for heat and mass transfer in presence of magnetic field over an inclined plate in the porous medium. Javaherdeh *et al.* (2015) performed numerical investigation to study the heat and mass transfer in MHD fluid flow past a moving vertical plate in a porous medium and presented that the Grashof number enhances the flow velocity while the porosity parameter reduces the velocity profile. Also, the rate of heat and mass transfer decreases with increase in transverse magnetic field. Sankar *et al.* (2006) have presented the effect of magnetic field on natural convection in a vertical cylindrical annulus. Their computational results reveal that in shallow cavities the flow and heat transfer are suppressed more effectively by an axial magnetic field while in tall cavities a radial magnetic field is more effective. Sheikholeslami *et al.* (2013) have done the numerical investigation of the effect of magnetic field on natural convection in a curved shape enclosure and concluded that Hartmann number can be a control parameter for heat and fluid flow. Singh *et al.* (1997) considered the study of steady fully developed laminar natural convection flow in open ended vertical concentric annuli in presence of radial magnetic field by considering the case of isothermal and constant heat flux on inner cylinder of concentric annuli. Furthermore, Singh and Singh (2012) have studied the effect of mixed kind of thermal boundary conditions on the free convective flow of an electrically conducting in annuli in the presence of a radial magnetic field. Jha *et al.* (2015) investigated the influence of externally applied transverse magnetic field on steady natural convection flow of conducting fluid in a vertical micro-channel.

The possibility of the viscous reactive fluid to exhibit apparent slip phenomenon through a vertical porous cylinder has received little attention, hence the thrust of the current research work area. The objective of this study therefore is to investigate the combined influence of Slip and Soret effects of a heat and mass transfer flow past a vertical porous cylinder in the presence of radial magnetic field. The governing equations along with the initial and boundary conditions are transformed into a dimensionless form and the resulting equations are solved by perturbation method. The closed form solutions for velocity, temperature, concentration, skin friction, Nusselt number as well as Sherwood number are presented. The effects of pertinent parameters on the flow, heat and mass transfer characteristics are discussed. To the best of my knowledge, this is the first time the results obtained in this research work will be reported in any literature.

**Mathematical analysis**

The geometry of the present paper under consideration is shown schematically in Fig. 1. We consider the steady state fully developed free convection flow of viscous reactive fluid with thermal diffusion and radial magnetic field in a vertical porous annulus of an infinite length as shown in figure 1. Immersed in a fluid at a constant temperature $T_0$, the fluid is assumed to be Newtonian and obeys Boussinesq's approximation. A uniform magnetic field $B_0$is applied perpendicularly to the direction of the flow. Under the above assumptions the concentration, energy and momentum equations in dimensional form are:

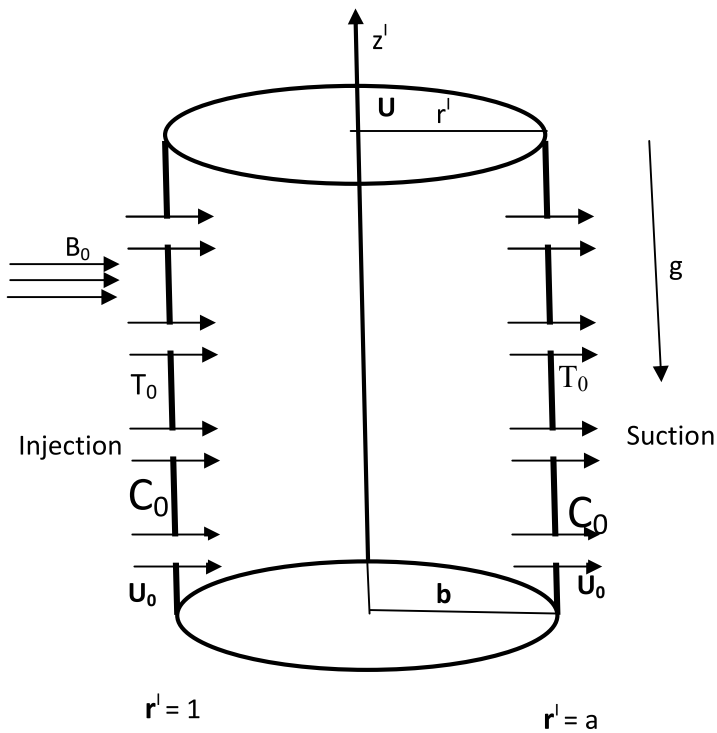


**Fig. 1 Physical coordinates of the flow problem**

$$\frac{\partial u'}{\partial t'}+\frac{v_0 b}{r'}\frac{\partial u'}{\partial r'}=\nu\left[\frac{\partial^2 u'}{\partial r'^2}+\frac{1}{r'}\frac{\partial u'}{\partial r'}\right]+g\beta\left(T'-T_0\right)+g\beta^*\left(\phi'-\phi_0\right)-\frac{\sigma\beta_0^2 b^*}{\rho r'}u' \tag{1}$$

$$\frac{\partial T'}{\partial t'}+\frac{v_0 b}{r'}\frac{\partial T'}{\partial r'}=\alpha\left[\frac{\partial^2 T'}{\partial r'^2}+\frac{1}{r'}\frac{\partial T'}{\partial r'}\right]+\frac{QC_0A}{\rho C_p}\exp\left(-\frac{E}{RT'}\right) \tag{2}$$

$$\frac{\partial \phi'}{\partial t'}+\frac{v_0 b}{r'}\frac{\partial \phi'}{\partial r'}=D_m\left[\frac{\partial^2 \phi'}{\partial r'^2}+\frac{1}{r'}\frac{\partial \phi'}{\partial r'}\right]+\frac{D_m K_T}{T_m}\left[\frac{\partial^2 T'}{\partial r'^2}+\frac{1}{r'}\frac{\partial T'}{\partial r'}\right] \tag{3}$$

The initial and boundary conditions to be satisfied are:

$$t\le 0: u'=P,\theta'=1,C'=1 \qquad for \qquad 1\le r'\le a$$

$$t'>0\begin{cases} u'=P,T'=T_w',C'=C_w' & at \quad r'=1 \\ u'=0 \quad \theta'=0 \quad C'=0 & at \quad r'=a \end{cases} \tag{4}$$

The physical quantities used in the above equations are defined in the nomenclature. It will be useful to obtain the solution of eqns, 1, 2 and 3 subject to eqn 4 in non-dimensional form. Therefore, we introduce the non-dimensional quantities:

$$u=\frac{u'}{U_0},t=\frac{t'\nu}{b^2},\theta=\frac{E}{RT_0^2}\left(T'-T_0\right),C=\frac{E\left(C'-C_0\right)}{RC_0^2},\lambda=\frac{QC_0AEb^2}{kRT_0^2}\exp\left(-\frac{E}{RT_0}\right),Sc=\frac{\nu}{D},$$

$$Sr=\frac{D_m K_T\left(T_1-T_0\right)}{\nu T_1\left(C_1-C_0\right)},Gr=\frac{\beta\left(T_1-T_0\right)}{\beta^*\left(C_1-C_0\right)},Gc=\frac{\beta^*\left(C_1-C_0\right)}{\beta\left(T_1-T_0\right)},M^2=\frac{\sigma B_0^2 r_0^2}{\rho\nu},\varepsilon=\frac{RT_0}{E},$$

$$r=\frac{r'}{b},s=\frac{v_0 b}{\nu} \tag{5}$$

**Analytical Solution**

Analytical solutions for the reactive viscous fluid are obtained for the concentration, temperature and velocity field as well as the skin friction, Nusselt number and Sherwood number. When the non-dimensional quantities are introduced, the governing equations of this problem in a closed-form are:

$$\frac{d^2u}{dr^2}+\frac{1}{r}(1-s)\frac{du}{dr}=\frac{M^2u}{r^2}-(Gr\theta+GcC) \tag{6}$$

$$\frac{d^2\theta}{dr^2}+\frac{1}{r}(1-s\Pr)\frac{d\theta}{dr}+\lambda\exp\left(\frac{\theta}{1+\varepsilon\theta}\right)=0 \tag{7}$$

$$\frac{d^2C}{dr^2}+\frac{1}{r}(1-sSc)\frac{dC}{dr}+Sr\left[\frac{d^2\theta}{dr^2}+\frac{1}{r}\frac{d\theta}{dr}\right]=0 \tag{8}$$

While the initial and boundary conditions are:

$$\begin{aligned} &u=P\frac{du}{dr}, \quad \theta=1, \quad C=1 \quad at \quad r=1 \\ &u=0, \quad \theta=0, \quad C=0 \quad at \quad r=a \end{aligned} \tag{9}$$

Where λ, P, Gr, Gc, M, s, Pr, Sc, Sr and εare Frank-Kamenetskii, Navier slip, Thermal Grashof number, solutal Grashof number, magnetic field, suction/injection, Prandlt number, Schmidt number, Soret number and activation energy parameters respectively. In order to construct approximate solutions to equations (6), (7) and (8) subject to (9), we employed a regular perturbation method by taking a power series expansion in the reactant consumption parameter $\lambda$

$$\phi=\sum_{i=0}^{\infty}\lambda^i\phi_i \tag{10}$$

$$\theta=\sum_{i=0}^{\infty}\lambda^i\theta_i \tag{11}$$

$$U=\sum_{i=0}^{\infty}\lambda^i u_i \tag{12}$$

substituting egns (10) (11) and (12) into eqn (6), (7) and (8) and equating the coefficients of like powers of $\lambda$, the solution of the governing equations for the momentum, energy and concentration fields subject to boundary conditions (9) are obtained as follows:

$$U=\lambda\left[B_3r^{z1}+B_4r^{-z2}+C_3r^2\right]+\lambda^2\left[B_5r^{z1}+B_6r^{-z2}+C_7r^4+C_8r^3+C_9r^2\right]+$$
$$\lambda^3\left[B_7r^{z1}+B_8r^{-z2}+C_{13}r^6+C_{14}r^5+C_{15}r^4+C_{16}r^3+C_{17}r^2\right] \quad (13)$$

$$\theta=f_4\left(v_1r+v_2\right)+\lambda\left[f_2\left(2v_3r+2v_4-r^2\right)\right]+\lambda^2\left[f_4f_2\left(\frac{r^4}{12}-\frac{v_3r^3}{3}-v_4r^2\right)+f_4\left(v_5r+v_6\right)\right]+$$
$$\lambda^3\left[\left\{f_4^2f_2\left(\frac{v_4r^4}{12}+\frac{v_3r^5}{60}-\frac{r^6}{360}\right)-f_4^2\left(\frac{v_5r^3}{6}+\frac{v_6r^2}{2}\right)\right\}+f_1f_3f_4\left(\frac{r^6}{30}-\frac{v_3r^5}{5}-\frac{v_4r^4}{3}+\right)\right.$$
$$\left.\left(\frac{v_3^2r^4}{3}+\frac{4v_3v_4r^3}{3}+2v_4^2r^2\right)+f_4\left(v_7r+v_8\right)\ \right] \quad (14)$$

$$\phi=-f_5Sr\left(f_4v_1\left(rIn\left(r\right)-r\right)\right)+f_5\left(v_9r+v_{10}\right)+\lambda\left[-f_5Sr\left(2f_2v_3\left(rIn\left(r\right)-r\right)-2f_2r^2\right)+f_5\left(v_{11}r+v_{12}\right)\right]+$$
$$\lambda^2\left[-f_5Sr\left\{f_4f_2\left(\frac{r^4}{9}-\frac{v_3r^3}{2}-2v_4r^2\right)+f_4v_5\left(rIn\left(r\right)-r\right)\right\}+v_5\left(v_{13}r+v_{14}\right)\right]+$$
$$\lambda^3\left[-f_5Sr\left\{f_4^2f_2\left(\frac{v_4r^4}{9}+\frac{v_3r^5}{48}-\frac{r^6}{300}\right)-f_4^2\left(\frac{v_5r^3}{4}+v_6r^2\right)\ \right\}+f_1f_3f_4\left(\frac{r^6}{25}-\frac{v_3r^5}{4}-\frac{4v_4r^4}{9}+\right.\right.$$
$$\left.\left.\frac{4v_3^2r^4}{9}+2v_3v_4r^3+4v_4^2r^2\right)+f_4v_7(rIn\left(r\right)-r)\ \right\}+f_5\left(v_{15}r+v_{16}\right)\ \Big] \quad (15)$$

The steady state skin frictions on the boundaries are:

$$\tau_1=\left.\frac{dU}{dr}\right|_{r=1}=\lambda\left(z1B_3-z2B_4+2C_3\right)+\lambda^2\left(z1B_5-z2B_6+4C_6+3C_7+2C_8\right)+$$
$$\lambda^3\left(z1B_7-z2B_8+6C_{11}+5C_{12}+4C_{13}+3C_{14}+2C_{15}\right) \quad (16)$$

$$\tau_b=\left.\frac{dU}{dr}\right|_{r=b}=\lambda\left(z1B_3b^{z1-1}-z2B_4b^{-z2-1}+2C_3b\right)+\lambda^2\left(z1B_5b^{z1-1}-z2B_6b^{-z2-1}+4C_6b^3+3C_7b^2+2C_8b\right)+$$
$$\lambda^3\left(z1B_7b^{z1-1}-z2B_8b^{-z2-1}+6C_{11}b^5+5C_{12}b^4+4C_{13}b^3+3C_{14}b^2+2C_{15}b\right) \quad (17)$$

Due to the symmetric condition of the flow, the mass and heat transfer on both boundaries is the same and therefore the steady state rate of heat and mass transfer at the boundaries are:

$$Nu = \left.\frac{d\theta}{dr}\right|_{r=b} = f_4 v_1 + \lambda\left[2 f_2\left(v_3 - b\right)\right] + \lambda^2\left[f_4 f_2\left(\frac{b^3}{3} - v_3 b^2 - 2v_4 b\right) + f_4 v_5\right] +$$
$$\lambda^3\left[f_4^2 f_2\left(\frac{v_4 b^3}{3} + \frac{v_3 b^4}{12} - \frac{b^5}{60}\right) - f_4^2\left(\frac{v_5 b^2}{2} + v_6 b\right) + f_1 f_3 f_4\left(\frac{b^5}{5} - v_3 r^4 - \frac{4v_4 b^4}{3} + \frac{4v_3^2 b^3}{3} -\right.\right.$$
$$\left.\left. 4 f_3 v_4 b^2 + 4v_4^2 b \right) + f_4 v_7 \right] \qquad (18)$$

$$Sh = \left.\frac{dC}{dr}\right|_{r=b} = \left[-f_5 f_4 v_1 Sr\left(In\left(b\right)\right)\right] + f_5 v_9 + \lambda\left[-f_5 Sr\left(2 f_2 v_3 In\left(b\right) - 4 f_2\right) + f_5 v_{11}\right] +$$
$$\lambda^2\left[-f_5 Sr\left\{f_4 f_2\left(\frac{4b^3}{9} - \frac{3v_3 b^2}{2} - 4v_4 b\right) + f_4 v_5 In\left(b\right)\right\} + f_5 v_{13}\right] + \lambda^3\left[f_5 Sr\left\{f_4^2 f_2\left(\frac{4v_4 b^3}{9} - \frac{5v_3 b^4}{48}\right.\right.\right.$$
$$\left. - \frac{b^5}{50}\right) - f_5 Sr\left(\frac{3v_5 b^2}{4} + 2v_6 b\right) + f_1 f_3 f_4\left(\frac{6b^5}{25} - \frac{5v_3 b^4}{4} - \frac{16v_4 b^3}{9} + \frac{16v_3^2 b^3}{9} + 6 v_3 v_4 b^2 + 8v_4^2 b\right)$$
$$\left. \left. + f_4 v_7 In\left(b\right) \right\} + f_5 v_{15} \right] \qquad (19)$$

*where* $f_1, f_2, f_3, f_4, f_5, v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, B_3, B_4, B_5$ $B_6, B_7, B_8, C_3, C_7, C_8, C_9, C_{13}, C_{14}, C_{15}, C_{16}, C_{17}$ *are defined* in *the appendix.*

**Table 1**:The numerical values of skin friction coefficients at both boundaries for Soret (Sr), thermal Grashof number (Gr), solutal Grashof number (Gc), Navier slip condition (P) and Hartmann number (M) against lambda ( $\lambda$ ) when s= 3,Sc=0.22 and Pr=0.71

| Sr | Gr | Gc | P | M | $\tau_1$ | $\tau_b$ |
|---|---|---|---|---|---|---|
| 1.0 | 1.0 | 1.0 | 1.0 | 0.01 | 0.0020 | -0.0081 |
| 2.0 | - | - | - | - | 0.0021 | -0.0085 |
| - | 5.0 | - | - | - | 0.0098 | -0.0390 |
| - | - | 5.0 | - | - | 0.0025 | -0.0097 |
| - | - | - | 2.0 | - | 0.0016 | -0.0100 |
| - | - | - | - | 1.0 | 0.0017 | -0.0072 |

**Table 2:** The numerical values of Nusselt number and Sherwood number for Sr, Sc, Pr against $\lambda$ when s=3

against

| Sc | Sr | Pr | Nu | Sh |
|---|---|---|---|---|
| 0.22 | 1.0 | 0.71 | 0.7905 | 1.3198 |
| 0.30 | - | - | - | 1.2625 |
| - | 2.0 | - | - | 1.5250 |
| - | - | 7 | -1.0000 | - |

## Results and discussion

The influence of Steady state free convection flow of viscous reactive fluids with Navier slip condition and Soret effects has been investigated in a vertical porous cylinder under radial magnetic field. In order to presents the effect of controlling parameters such as Frank – Kamenetskii ($\lambda$), Slip parameter (P), Thermal Grashof number (Gr), Solutal Grashof number (Gc), Soret number (Sr) and Hartmann number (M) on the flow, we have presented the non-dimensional momentum, temperature, concentration as well as skin friction, Nusselt number and Sherwood number in Fig. 2-11. Fig. 2 shows the effect of $\lambda$ on velocity. It is observed from this figure that, increase in $\lambda$, causes a significant enhancement in the velocity of the fluid. This is due to the fact that, increasing $\lambda$ corresponds to the increase in the strength of the chemical reaction. The influence of the slip parameter on the fluid motion is demonstrated in Fig. 3. It is revealed that, the fluid velocity at the lower boundary surface increases with an increase in P. As expected, higher values of P means an increase in the reaction and slipperiness of the lower channel. It is clear from figure 4 that, increasing values of Sr, increases the motion of the fluid. From Fig. 5and 6, the effect of thermal Grashof number and solutal Grashof number on velocity profile respectively is shown. It is evident from these figures that increasing values of Gr and Gc, leads to an increase in the velocity of the fluid. Fig. 7 presented the effect of M on velocity profile. It is obvious from this figure that, increasing M parameters, suppresses the fluid flow significantly. Physically, $M$ =0 means that there is no magnetic effect and the flow is purely hydrodynamic. It is found from this Figure that the velocity decreases with increasing value of $M$. This is expected as the application of the transverse magnetic field always results in a resistance-type force called Lorentz force. This force is similar in nature to a drag force, and upon increasing the values of $M$ the drag force increases and tends to resist the fluid flow, thus reducing the fluid motion significantly. Similarly, increase in Pr, leads to an improvement in the velocity flow depicted in Fig. 8. Fig. 9 and Fig. 10 show the effect of $\lambda$ on temperature and concentration and velocity respectively. It is observed from these figures that, increase in $\lambda$, increases the temperature and concentration of the fluid. The variation of Schmidt number on the concentration of the fluid is shown in Fig. 11. It is abundantly clear from the figure that, the concentration distribution suffers a substantial decrease faster as the diffusing foreign species becomes heavier. Thus, higher Sc leads to a suppression of the flow pattern.

The computational results of skin friction coefficients at both boundaries for varying Sr, Gr, Gc, P and M against λ when s=3, Sc=0.22 and Pr=0.71 is illustrated in

values of

Table 1. It is revealed from the table that, Sr, Gr and Gc are noticed to increase the shear stress at the lower boundary where r=1, while the reverse situation occurred at the upper boundary as seen from table 1. Also, the Navier slip parameter and magnetic field suppresses the skin friction for increasing values respectively at the lower boundary while the opposite phenomenon is observed at the other boundary. Table 2 shows the characteristics behavior of Pr, Sc and Sr for Nusselt number and Sherwood number. It is clearly seen from the table that, as the rate of mass transfer is noticed to be intensifying for increasing values of Sr, the reverse results is recorded for increasing values of Sc number. The rate of heat transfer coefficient is found to decrease for accelerating values of Pr as revealed in Table 2.

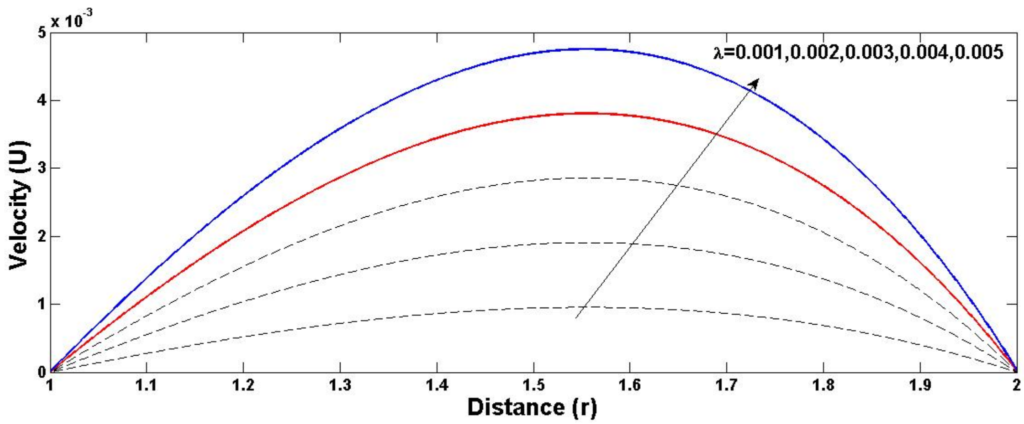


**Fig. 2 Effect of λ on velocity profile (for ε=0.01,Pr=0.71, s=3, Sc=0.22)**

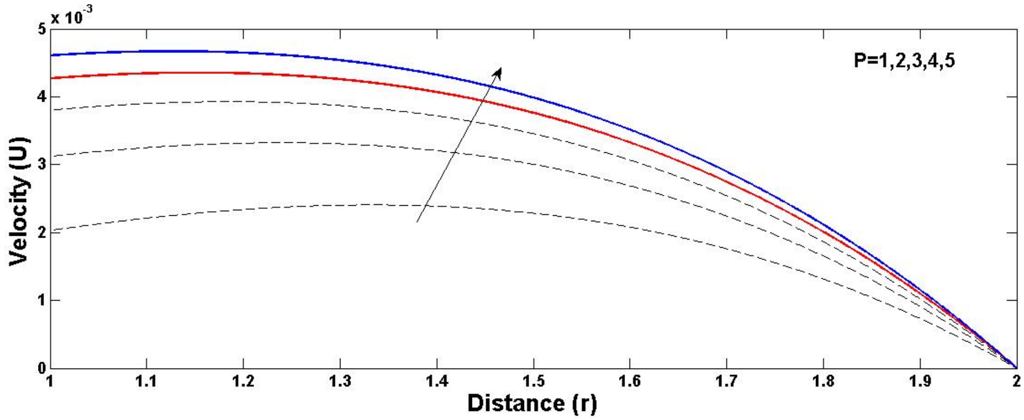


**Fig. 3 Effect of slip parameter on velocity profile (for λ =0.001, ε=0.01,Pr=0.71, s=3, Sc=0.22)**

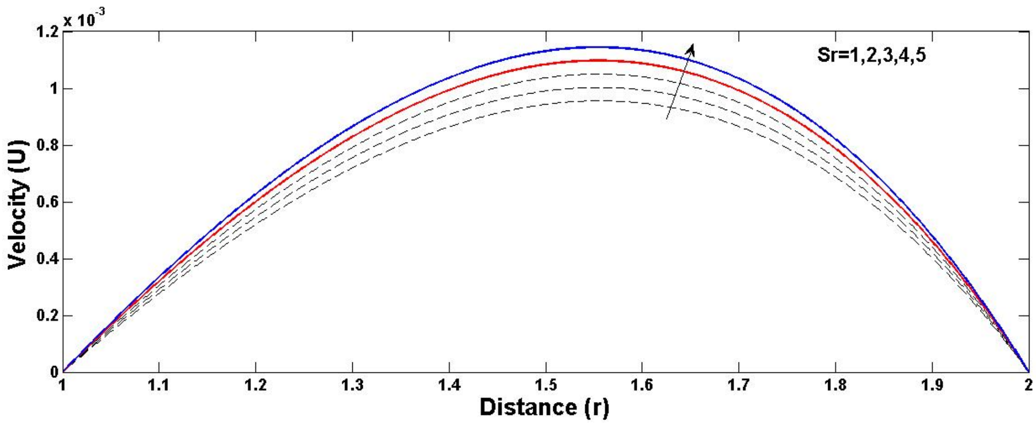


**Fig. 4 Effect of Soret on velocity profile (for λ =0.001, ε=0.01,Pr=0.71, s=3, Sc=0.22)**

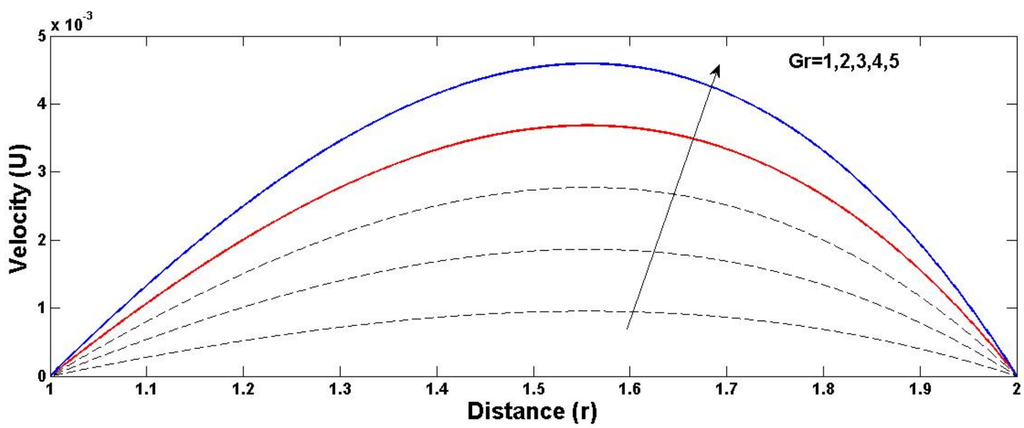


**Fig. 5 Effect of thermal Grashof number on velocity profile (forλ =0.001, ε=0.01,Pr=0.71, s=3, Sc=0.22)**

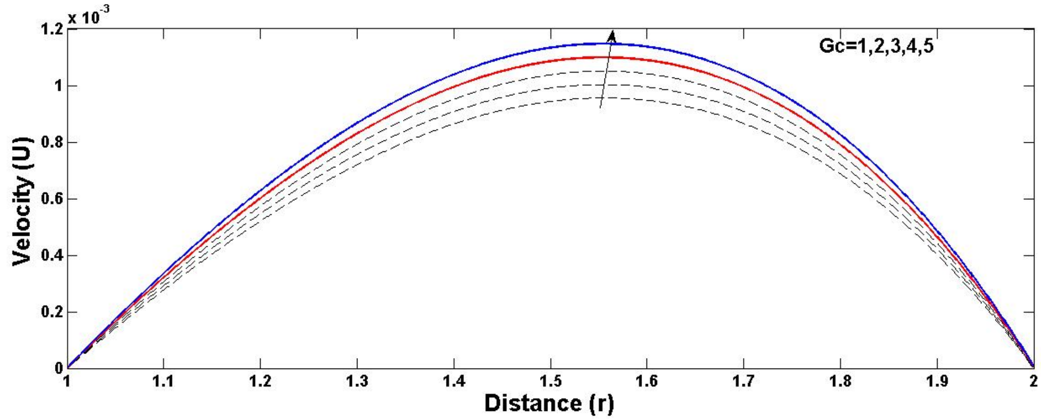


**Fig. 6 Effect of Solutal Grashof number on velocity profile (forλ =0.001, ε=0.01,Pr=0.71,s=3, Sc=0.22)**

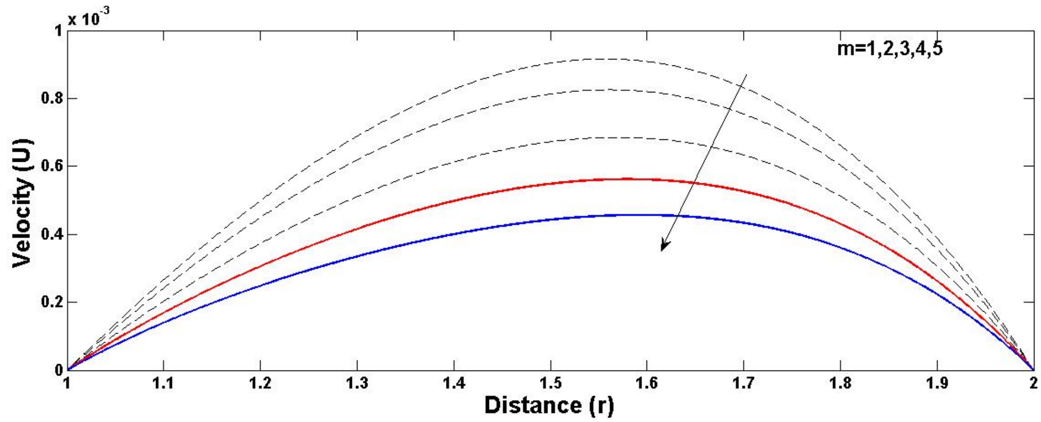


**Fig. 7 Effect of magnetic field on velocity profile (forλ =0.001, ε=0.01,Pr=0.71, s=3, Sc=0.22)**

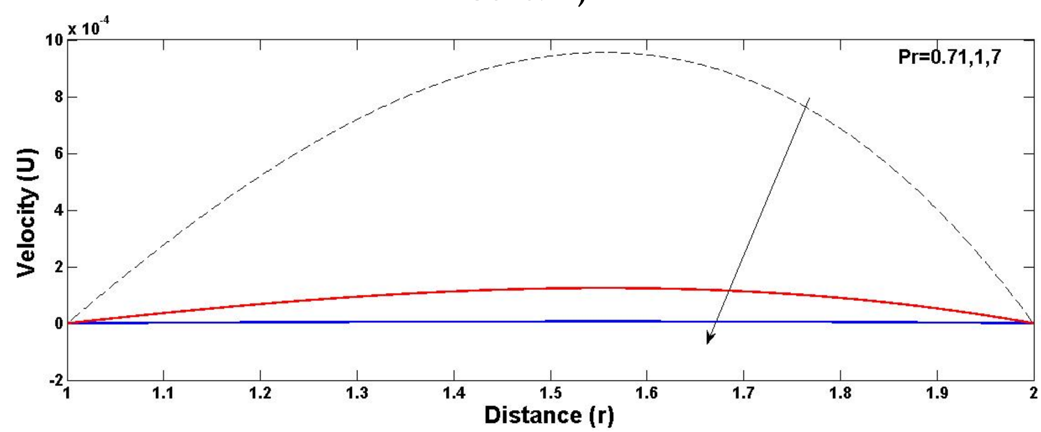


**Fig. 8 Effect of Prandlt number on velocity profile (forλ =0.001, ε=0.01, s=3, Sc=0.22)**

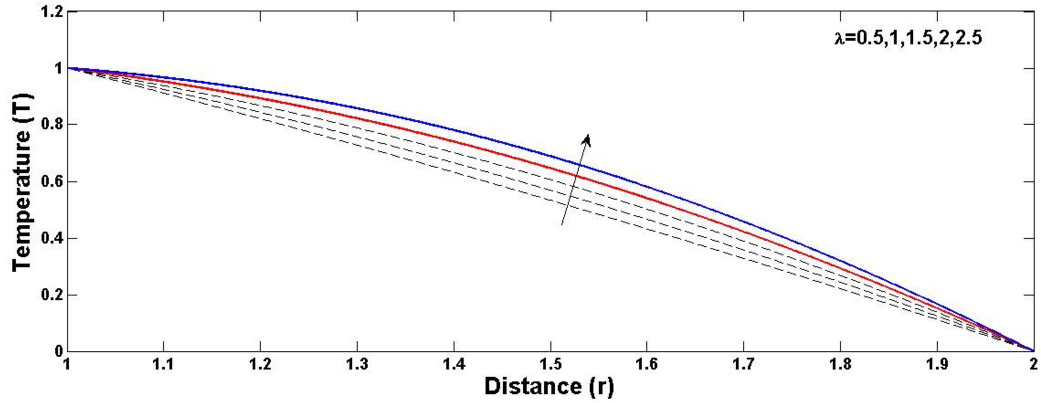


Fig. 9 Effect of λ on temperature profile (for ε=0.01,Pr=0.71, s=3 )

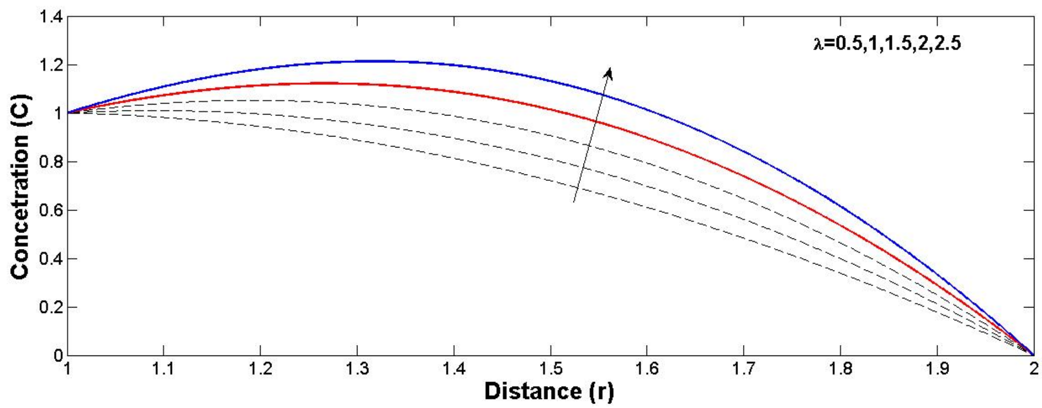


Fig. 10 Effect of λ on concentration profile (for s=3, Sc=0.22)

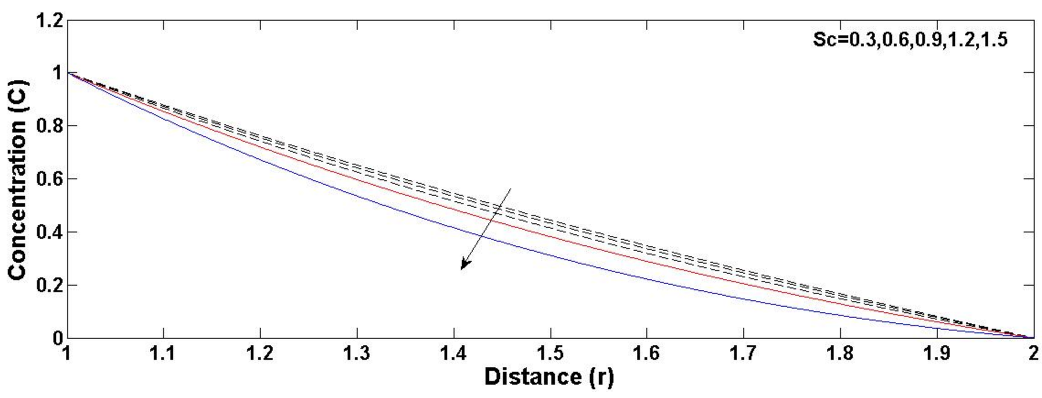


Fig. 11 Effect of Schmidt number on concentration profile (forλ =0.001, ε=0.01,Pr=0.71, s=3)

## Conclusion

The problem of free convection flow of viscous reactive fluids with the combined effects of Navier slip condition and Soret from a vertical porous cylinder has been presented under external radial magnetic effect. The non-linear governing equations are solved analytically by employing the perturbation series method. The influence of varying values of Frank – Kamenetskii ($\lambda$), Navier slip condition (P), Soret effect (Sr), thermal Grashof number (Gr), solutal Grashof number (Gc) and magnetic field (m) on the flow formation are discussed. The result of skin friction, Nusselt number and Sherwood number are also demonstrated by means of tables. It is interesting to report that, increasing values of Frank – Kamenetskii ($\lambda$), Slip parameter (P), thermal Grashof number (Gr), Solutal Grashof number (Gc), Soret number (Sr) increases the velocity while increasing the radial magnetic field and Prandlt number decreases the fluid flow consequently causing the skin friction to behave in a pattern that obeys natural phenomenon as demonstrated in Table 1. Also, as $\lambda$ increases, the temperature and concentration gradients is enhanced leading to an increase in the rate of heat and mass transfer flow as elucidated in Table 2

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*Appendix*

$$f_1 = e-\frac{1}{2}, f_2 = \frac{1}{4-2s\Pr}, f_3 = \frac{1}{(2-s\Pr)^2}, f_4 = \frac{1}{2-s\Pr}, f_5 = \frac{1}{2-sSc}, z_1 = \frac{s+\sqrt{s^2+4M^2}}{2},$$

$$z_2 = \frac{-s+\sqrt{s^2+4M^2}}{2}, v_1 = \frac{2-s\Pr}{1-b}, v_2 = -v_1 b, v_3 = \frac{b^2-1}{2(b-1)}, v_4 = \frac{b^2}{2} - v_3 b,$$

$$v_5 = \frac{x_2}{b-1}\left\{ v_4\left(b^2-1\right) + \frac{v_3\left(b^3-1\right)}{3} + \frac{\left(1-b^4\right)}{12}\right\}, v_6 = f_2\left(v_4 b^2 + \frac{v_3 b^3}{3} - \frac{b^4}{12}\right) - v_5 b,$$

$$v_7 = \frac{1}{b-1}\left[ f_2\left\{ f_2\left(\frac{b^6-1}{360}\right) + v_3\left(\frac{1-b^5}{60}\right) + v_4\left(\frac{1-b^4}{12}\right) + \right.\right.$$

$$\left.\left.\frac{v_5\left(b^3-1\right)}{6} + \frac{v_6\left(b^2-1\right)}{2} \right\} + f_1 f_3\left(\frac{1-b^6}{30} + \frac{v_3\left(b^5-1\right)}{5} + \frac{v_4\left(b^4-1\right)}{3} + \frac{v_3^2\left(1-b^4\right)}{3} + \frac{4v_3v_4\left(1-b^3\right)}{3} + 2v_4^2\left(1-b^2\right)\right)\right]$$

$$v_8 = \left[ f_2\left\{ f_2\left(\frac{b^6}{360}\right) - v_3\frac{b^5}{60} - v_4\frac{b^4}{12} + \frac{v_5 b^3}{6} + \frac{v_6 b^2}{2} \right\} - f_1 f_3\left(\frac{b^6}{30} - \frac{v_3 b^5}{5} - \frac{v_4 b^4}{3} + \frac{v_3^2 b^4}{3} + \frac{4v_3v_4b^3}{3} + 2v_4^2 b^2\right) - v_7 b\right]$$

$$v_9 = \frac{Sr}{b-1}\left\{ f_4 v_1\left(1+b\log b - b\right)\right\} - \frac{2+sSc}{b-1}, v_{10} = Sr\left\{ f_4 v_1\left(1+b\log b-b\right)\right\} - v_9 b$$

$$v_{11} = \frac{Sr}{2-sSc}\left\{2f_2v_3\left(1+b\log b-b\right) + 2f_2\left(1-b^2\right)\right\}, v_{12} = Sr\left\{2f_2v_3\left(1+b\log b-b\right)+2f_2\left(1-b^2\right)\right\} - v_{11}b$$

$$v_{13} = \frac{Sr}{b-1}\left[ f_4 f_2\left(\frac{b^4-1}{9} + \frac{v_3\left(1-b^3\right)}{2} + 2v_4\left(1-b^2\right) + f_4 v_5\left(1+b\log b - b\right)\right)\right]$$

$$v_{14} = Sr\left[ f_4 f_2\left(\frac{b^4}{9} + \frac{v_3 b^3}{2} + 2v_4 b^2 + f_4 v_5\left(b\log b - b\right)\right)\right] - v_{13} b$$

$$v_{15} = \frac{Sr}{b-1}\left[ f_4^2\left\{ f_2\left(\frac{v_4\left(1-b^4\right)}{9} + \frac{5v_3\left(1-b^5\right)}{48} + \frac{b^6-1}{300}\right) - v_5\frac{\left(1-b^3\right)}{6} - v_6\left(1-b^2\right)\right.\right.$$

$$f_1 f_3 f_4\left(\frac{1-b^6}{50} + \frac{v_3\left(b^5-1\right)}{4} + \frac{4v_4\left(b^4-1\right)}{9} + \frac{4v_3^2\left(1-b^4\right)}{9} + 2v_3v_4\left(1-b^3\right) + 4v_4^2\left(1-b^2\right)\right)$$

$$\left.\left. f_4 v_7 (1+b\log b - b) \right\}\right]$$

$$v_{16} = Sr\left[ f_4^2\left\{ f_2\left(\frac{v_4 b^4}{9} + \frac{5v_3 b^5}{48} - \frac{b^6}{300}\right) - v_5\frac{b^3}{4} - v_6 b^2\right.\right.$$

$$f_1 f_3 f_4\left(\frac{2b^6}{50} - \frac{v_3 b^5}{4} - \frac{4v_4\left(b^4\right)}{9} + \frac{4v_3^2\left(b^4\right)}{9} + 2v_3v_4\left(b^3\right) + 4v_4^2\left(b^2\right)\right)$$

$$\left.\left. f_4 v_7 (1+b\log b - b) \right\}\right] - v_{15} b$$

$$\eta_8 = -2x_2k_4, \eta_{12} = -Nx_5k_{12}, \eta_{15} = x_2x_4k_4, \eta_{16} = -x_4k_5, \eta_{17} = -x_4k_6, \eta_{20} = -NSrx_2x_4x_5k_4, \eta_{24} = -NSrx_4x_5k_5$$

$$\eta_{25} = -Nx_5k_{13}, \eta_{26} = -Nx_5k_{14}, \eta_{27} = \frac{-x_4^2x_2k_4}{12}, \eta_{30} = \frac{-x_4^2k_5}{12}, \eta_{31} = \frac{-x_4^2k_6}{2}, \eta_{32} = x_1x_3x_4, \eta_{35} = \frac{-\eta_{32}k_4}{3},$$

$$\eta_{36} = \frac{\eta_{32}k_3^2}{3}, \eta_{37} = \frac{-4\eta_{32}k_3k_4}{3}, \eta_{38} = 2\eta_{32}k_4^2, \eta_{39} = -x_4k_7, \eta_{40} = -x_4k_8, \eta_{41} = \frac{-NSrx_4^2x_2x_5k_4}{12},$$

$$\eta_{44} = \frac{-NSrx_5x_4^2k_5}{6}, \eta_{45} = \frac{-NSrx_5x_4^2k_6}{2}, \eta_{48} = N\eta_{35}Srx_5, \eta_{49} = N\eta_{36}Srx_5, \eta_{50} = N\eta_{37}Srx_5,$$

$$\eta_{51} = N\eta_{38}Srx_5, \eta_{52} = \frac{-NSrx_4^2x_2x_5k_4}{36}, \eta_{55} = \frac{-NSrx_4^2x_5k_5}{12}, \eta_{56} = \frac{-NSrx_4^2x_5k_6}{2}, \eta_{60} = \frac{N\eta_{32}Srx_5k_3^2}{9},$$

$$\eta_{61} = \frac{2NSrx_5k_3k_4}{2}, \eta_{62} = 2\eta_{32}NSrx_5k_4^2, \eta_{63} = NSrx_5x_4k_7\left(\log(r) - r\right), \eta_{64} = -Nx_5k_{15}, \eta_{65} = -Nx_5k_{16}$$

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$$C_3 = \frac{\eta_8 + \eta_{12}}{2 + 2(1-s) - M^2}, C_7 = \frac{\eta_{15} + 2\eta_{20}}{12 + 4(1-s) - M^2}, C_8 = \frac{\eta_{16} + \eta_{24} + \eta_{25}}{6 + 3(1-s) - M^2}$$

$$C_9 = \frac{\eta_{16} + \eta_{26}}{2 + 2(1-s) - M^2}, C_{13} = \frac{\eta_{27} + \eta_{35} + \eta_{36} + \eta_{41} + \eta_{48} + \eta_{49} + \eta_{52} + \eta_{60}}{30 + 6(1-s) - M^2}$$

$$C_{14} = \frac{\eta_{30} + \eta_{37} + \eta_{44} + \eta_{50} + \eta_{55} + \eta_{61}}{20 + 5(1-s) - M^2}, C_{15} = \frac{\eta_{31} + \eta_{38} + \eta_{45} + \eta_{51} + \eta_{56} + \eta_{62}}{12 + 4(1-s) - M^2}$$

$$C_{16} = \frac{\eta_{39} + \eta_{63} + \eta_{64}}{6 + 3(1-s) - M^2}, C_{17} = \frac{\eta_{40} + \eta_{65}}{2 + 2(1-s) - M^2}, B_3 = \frac{C_3(2P-1) - B_4(z_2P+1)}{(1-z_1P)},$$

$$B_4 = \frac{C_3b^{z_1}(2P-1) + C_3b^2(1-z_1P)}{b^{z_1}(z_2P+1) - b^{-z_2}(1-z_1P)}, B_5 = \frac{C_6(4P-1) - B_6(z_2P+1) + C_7(3P-1) + C_8(2P-1)}{(1-z_1P)},$$

$$B_6 = \frac{C_6\left\{b^{z_1}(4P-1) + b^4(1-z_1P)\right\} + C_7\left\{b^{z_1}(3P-1) + b^3(1-z_1P)\right\} + C_8\left\{b^{z_1}(2P-1) + b^2(1-z_1P)\right\}}{b^{z_1}(z_2P+1) - b^{-z_2}(1-z_1P)},$$

$$B_7 = \frac{C_{11}(6P-1) - B_8(z_2P+1) + C_{12}(5P-1) + C_{13}(4P-1) + C_{14}(3P-1) + C_{15}(2P-1)}{(1-z_1P)},$$

$$B_8 = \frac{\begin{array}{c} C_{11}\left\{b^{z_1}(6P-1) + b^6(1-z_1P)\right\} + C_{12}\left\{b^{z_1}(5P-1) + b^5(1-z_1P)\right\} + C_{13}\left\{b^{z_1}(4P-1) + b^4(1-z_1P)\right\} + \\ C_{14}\left\{b^{z_1}(3P-1) + b^3(1-z_1P)\right\} + C_{15}\left\{b^{z_1}(2P-1) + b^2(1-z_1P)\right\} \end{array}}{b^{z_1}(z_2P+1) - b^{-z_2}(1-z_1P)}$$


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