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Research Article

Mathematical Analysis of Chickenpox Transmission Dynamics with Control Measures

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Abstract

In this work, we developed and analyzed a mathematical model for the control of chickenpox with vaccination and treatment strategy. The model is a first order Ordinary Differential Equations, in which the human population is divided into five compartments namely; Susceptible individuals (S), Vaccinated individuals (V), Infected individuals (I), Treated individuals (T) and Removed individuals (R).The equilibrium states were obtained and their stabilities were analyzed. The result shows that the disease free equilibrium state is stable and the criteria for stability of the endemic equilibrium state are established.

Keywords: Chickenpox, Vaccination, Treatment, Modeling.

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Introduction

Currently, fear of infectious disease is a great challenge to human as some of these diseases lead to termination of life (death). It becomes necessary to pay enormous attention to control and prevention of these diseases. Infectious diseases could be Lassa fever, smallpox, cholera etc and varicella which is the subject matter of this research work.

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Chickenpox, also known as varicella, is a highly infectious disease caused by the initial infection with *Varicella zoster* virus (VZV) (CDC., 2011a). Chickenpox is an airborne disease which spreads easily through the coughs and sneezes of an infected person (CDC., 2011b). It may be spread from one to two days before the rash appears until all lesions have crusted over (CDC., 2011c). It may also spread through contact with the blisters (CDC., 2011b).The disease results in a characteristic skin rash that forms small, itchy blisters, which eventually scab over (Atkinson and William, 2011). It usually starts on the chest, back, and face then spreads to the rest of the body. Other symptoms may include fever, tiredness, and headaches (Atkinson and William, 2011). Symptoms usually last five to seven days (Atkinson and William, 2011). Complications may occasionally include pneumonia, inflammation of the brain, and bacterial skin infections (Garnett and Greenfell,1992). The disease is often more severe in adults than in children (Garnett and Greenfell, 1992). Symptoms begin 10 to 21 days after exposure to the virus (CDC., 2011a).

The varicella vaccine protects about 70 to 90 percent of people from disease with a greater benefit for severe disease (Garnett and Greenfell, 1992). Routine immunization of children is recommended in many countries (Leeuwen, 2015). Immunization within three days of exposure may improve outcomes in children (Nthiiri *et al*., 2015). Treatment of those infected may include calamine lotion to help with itching, keeping the fingernails short to decrease injury from scratching, and the use of paracetamol (acetaminophen) to help with fevers (CDC., 2011b).Chickenpox occurs in all parts of the world (Garnett and Greenfell, 1992). In 2013 there were 140 million cases of chickenpox and herpes zoster worldwide ((Nthiiri *et al*., 2015). In 2015 chickenpox resulted in 6,400 deaths globally – down from 8,900 in 1990 (Dieman *et al*., 1990; Wang *et al*., 2005).

The varicella vaccine is recommended in many countries (Leeuwen, 2015). Some countries require the varicella vaccination or an exemption before entering elementary school. A second dose is recommended five years after the initial immunization (Dieman *et al*. 1990). A vaccinated person is likely to have a milder case of chickenpox if they become infected (Garnett and Greenfell, 1992). Immunization within three days following household contact reduces infection rates and severity in children (Nthiiri *et al*., 2015).

In populations that have not been immunized or if immunity is questionable, a clinician may order an Enzyme immunoassay. An immunoassay measures the levels of antibodies against the virus that give immunity to a person. If the levels of antibodies are low (low titer) or questionable, re-immunization may be done (CDC., 2011a). Treatment mainly consists of easing the symptoms. As a protective measure,

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people are usually required to stay at home while they are infectious to avoid spreading the disease to others.

Over the years, different authors have used different approaches to model the disease. For instance, Stephen *et al*. (2014) developed and carried out stability analysis for a *Varicella zoster* virus model with vaccination. Donmarin *et al*., (2008), modeled the impact of vaccination on the epidemiology of *Varicella zoster* virus.

Wang *et al*. (2005) studied Susceptible Infected Recovered (SIR) epidemic model with age structure: using integral – differential equation and a fixed point. Henry *et al*. (2006) investigated the properties of a simple discrete time stochastic Epidemic model. Stephen *et al*. (2014) also developed and analyzed a model for a *Varicella zoster* virus model with vaccination.

In this article, we propose a new mathematical model for Chickenpox with vaccination and treatment strategies to handle the menace of Chicken pox in any given population. All aspects of Chicken pox transmission dynamics were incorporated to come up with a mathematical model of Chicken pox.

Materials and Methods

The total population is divided into the following epidemiological stages: susceptible **(S)**, Vaccinated **(V)**, Infected **(I)**, Treated **(T)**, Removed **(R).**

Let λ be the constant recruitment rate, where the susceptible individuals are recruited by both birth and immigration. Furthermore, we assume that individuals susceptible can be vaccinated through direct contact b, and that the vaccinated individual can also be susceptible due to vaccine failure through the contact rate a.

Let β be the probability that the susceptible individual becomes infected by an infectious individual, the infected individual enters the treated compartment by rate α, in the treated population μ and m are the natural death rate and death due to infection during treatment.ω is the probability that treated individual becomes removed and n death due to natural immunity. Base on the nature of the diesease, the removed individual are permanently immured and cannot be susceptible to it any more.

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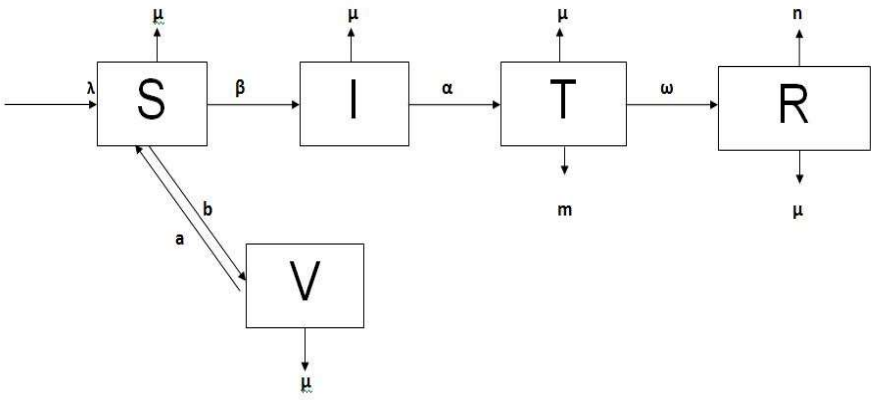

Figure 1.The diagram for the flow of the model

Basic Assumptions;

- The population is fixed.
- Susceptible individual that are vaccinated can also be susceptible due to vaccine failure.
- there is permanent immunity.
- The population mixture is homogeneous.
- Sex, age, race and social status do not affect the probability of individual being infected.
- there is no inherited immunity.

From the mathematical assumptions and figure 1, we have the following differential equations:

$$\frac{dS}{dt} = \lambda + bV - (a + \mu + \beta)S \qquad (2.1)$$

$$\frac{dV}{dt} = aS - (b + \mu)V \qquad (2.2)$$

$$\frac{dI}{dt} = \beta S - (\mu + \alpha)I \qquad (2.3)$$

$$\frac{dT}{dt} = \alpha I - (\mu + m + \omega)T \qquad (2.4)$$

$$\frac{dR}{dt} = \omega T - (n + \mu)R \qquad (2.5)$$

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where $\beta = \frac{I}{N}$

Description of the variables and parameters used in our model is tabulated in Table 1.

TABLE 1. STATE VARIABLES

VARIABLE	DESCRIPTION
S(t)	Susceptible individual at time (t)
V(t)	vaccinated human at time (t)
I(t)	infected individual at time (t)
T(t)	treated human at time (t)
R(t)	Removed human at time (t)
N(t)	the total population at time (t)
λ	Recruitment rate into susceptible individual
β	contact rate of infectious
α	the rate at which infected human becomes treated
ω	the rate at which treated individual becomes removed
μ	natural death rate
M	death due to infection during treatment
A	rate at which the susceptible human becomes vaccinated
B	rate at which vaccinated human becomes susceptible due to vaccine failure
N	death due to natural immunity

Invariant Region

The total population at time t is:

$$N(t) = V + S + I + T + R \quad (2.6)$$

$$\frac{dN}{dt} = \frac{dV}{dt} + \frac{dS}{dt} + \frac{dI}{dt} + \frac{dT}{dt} + \frac{dR}{dt} \quad (2.7)$$

$$\frac{dN}{dt} = \lambda - \mu V - \mu S - \mu I - \mu I - \mu T - mT - \mu R - nR \quad (2.8)$$

The invariant region can be shown by using the following theorem.

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Theorem 1

The solutions of the model equation are feasible for all $t>0$, if they enter the invariant region Ω

proof :

Let

$\Omega = (V, S, I, T, R) \varepsilon R_{+}^{5}$

be any solution of the model equations with non negative initial conditions.

suppose there is no disease induced death equation (2.8) becomes

$$\frac{dN}{dt} \leq \lambda - \mu S \quad (2.9)$$

$$\Rightarrow \frac{dN}{dt} \leq \lambda - \mu N \quad (2.10)$$

I.e

$S = N.$

$$\frac{dN}{dt} + \mu N \leq \lambda \quad (2.11)$$

The integrating factor of equation (11) is $e^{\mu t}$

$$\frac{dN}{dt}(e^{\mu t}) + \mu N (e^{\mu t}) \leq \lambda e^{\mu t} \quad (2.12)$$

$$\frac{dN}{dt}(Ne^{\mu t}) \leq \lambda e^{\mu t} \quad (2.13)$$

$$d(Ne^{\mu t}) \leq \lambda e^{\mu t} dt \quad (2.14)$$

integrating both sides and applying the initial condition, we have

$t(0) = N(0)$

$$N(t) \leq \frac{\lambda}{\mu} + (N(0) - \frac{\lambda}{\mu}) e^{-\mu t} \quad (2.15)$$

Therefore the human population approaching the carrying capacity $\frac{\lambda}{\mu}$ as $t \rightarrow \infty$

Hence, the feasible solutions set of the model equation enter the region

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$$\{\Omega = (V, S, I, T, R)\varepsilon R_+^5 / V > 0,\ S > 0,\ I \geq 0,\ T \geq 0,\ R \geq 0,\ N < \frac{\lambda}{\mu}\}$$

Thus the model is biologically feasible.

Hence whenever , $N > \frac{\lambda}{\mu}$, then $N < 0$, which means the population reduces asymptotically to the carrying capacity and whenever , $N \leq \frac{\lambda}{\mu}$ every solution with initial condition in Ω, therefore positively invariant (i.e solutions remains positive for all time $t > 0$) and the model is well posed and biologically meaningful.

Positivity of solutions

Theorem 2

Let the initial data be $\{S(0) > 0, V(0), I(0), T(0), R(0) \geq 0\}$

Then the solution set $\{V, S, I, T, R\}(t)$ of the system of equations 1 to 5 is positive for all $t > 0$

proof

from equation (1)

$$\frac{ds}{dt} = \lambda + bv - (a + \mu + \beta)S \leq -(a + \mu + \beta)S \quad (2.16)$$

$$\frac{ds}{dt} \leq -(a + \mu + \beta)S \quad (2.17)$$

Integrating and applying the initial condition and applying the initial conditions t=0, $S(0) \leq k_1$

We have;

$$S(t) \leq S(0)e^{-(a+\mu+\beta)t} \quad (2.18)$$

from equation 2

$$\frac{dV}{dt} = aS - (b - \mu)V \leq -(b + \mu)V \quad (2.19)$$

$$\Rightarrow \frac{dV}{dt} \leq -(b + \mu)V \quad (2.20)$$

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Integrating and applying the initial condition and applying the initial conditions t= 0, $V(0) = k_2$

We have; $V(t) \leq V(0)e^{-(b+\mu)t}$ (2.21)

Similarly, it can be verified that the rest of the equations are positive for all $t > 0$ since $e^{\omega} > 0$ for all $\omega \in \square$

Equilibrium States

The Disease Free Equilibrium State of the Model

At disease free Equilibrium any class that has infection will be considered as zero.

Therefore the disease free Equilibrium is

$$(S^*, V^*, I^*, T^*, R^*) = [\frac{(b+\mu)\lambda}{(a+\mu)(b+\mu)-ab}, \frac{a\lambda}{(a+\mu)(b+\mu)-ab}, 0, 0, 0]$$

The Endemic Equilibrium State of the Model

The Equilibrium state with the presence of infection is known as endemic Equilibrium state.

The endemic Equilibrium states of our model equation 1 to 5 are given below:

$$S^* = N(\mu+\alpha)$$

$$V^* = \frac{aN\ (\mu\ +\ \alpha\ )}{b\ +\ \mu}$$

$$I^* = \frac{\lambda}{(\mu+\alpha)} + \frac{abN}{b+\mu} - aN - \mu N$$

$$T^* = \frac{\alpha}{u+m+\omega}(\frac{\lambda}{(\mu+\alpha)} + \frac{abN}{b+\mu} - aN - \mu N)$$

$$R^* = \frac{\alpha w}{(n+\mu)(u+m+\omega)}(\frac{\lambda}{(\mu+\alpha)} + \frac{abN}{b+\mu} - aN - \mu N)$$

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The Reproduction Number

One of the most vital concern about any infectious diseases is its ability to invade a population. The basic reproduction number R_0 is a measure of the potential for disease spread in a population and is inarguably one of the foremost and most valuable ideas that mathematical thinking has brought to epidemic theory. According to [17], it represents the average number of secondary cases generated by an infected individual. If introduced into susceptible population with no immunity to the disease in the absence of intervention to control of the infection, if $R_0 < 1$. Then an average infected individual produces less than one newly infected individual over the course of an infection period, in this case the disease may die out in the long run. On the other hand if $R_0 > 1$, each infected individual produces an average more than one new infection, the infection will be able to spread in a population.

By using the next generation matrix

$$f_i = \begin{pmatrix} \frac{I}{N} \\ \omega T \end{pmatrix} \qquad \text{..... ....}(+)$$

$$V_i = \begin{pmatrix} (\mu+\alpha)I \\ (n+\mu)R \end{pmatrix} \qquad \text{.....(++)}$$

Differentiating f_i with respect to I and R we have

$$F = \begin{pmatrix} \frac{1}{N} & 0 \\ 0 & 0 \end{pmatrix}$$

Differentiating V_i with respect to I and R we have

$$V = \begin{pmatrix} (\mu+\alpha) & 0 \\ 0 & (n+\mu) \end{pmatrix}$$

the next generation matrix FV^{-1} is given by

$$FV^{-1} = \begin{pmatrix} \frac{1}{N} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{(\mu+\alpha)} & 0 \\ 0 & \frac{1}{(n+\mu)} \end{pmatrix} = \begin{pmatrix} \frac{1}{N(\mu+\alpha)} & 0 \\ 0 & 0 \end{pmatrix}$$

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R_0 is the spectra radius of $|FV^{-1} - I\lambda| = 0$

Therefore, the basic reproduction number $R_0 = \rho(FV^{-1}) =$ spectra radius of FV^{-1}

$R_0 = \dfrac{1}{N(\mu + \alpha)}$ since is the largest eigenvalue.

Local Stability Analysis of Disease Free Equilibrium State

To obtain the local stability analysis of the disease free equilibrium state, we recall that the system of Equilibrium of the model at Equilibrium is

$$A = \lambda + bV - (a + \mu + \beta)S$$
$$\frac{\partial A}{\partial I} = \frac{\partial A}{\partial T} = \frac{\partial A}{\partial R} = 0$$
$$B = as - (b + \mu)V = 0$$
$$\frac{\partial B}{\partial S} = a, \frac{\partial B}{\partial V} = -(b+\mu), \frac{\partial B}{\partial I} = \frac{\partial B}{\partial T} = \frac{\partial B}{\partial R} = 0$$
$$C = \beta S - (\mu + \alpha) I = 0$$
$$\frac{\partial C}{\partial S} = \beta, \frac{\partial C}{\partial V} = 0, \frac{\partial C}{\partial I} = -(\mu+\alpha), \frac{\partial C}{\partial T}, \frac{\partial C}{\partial R} = 0$$
$$D = \alpha I - (u + m + w) = 0$$
$$\frac{\partial D}{\partial S} = 0, \frac{\partial D}{\partial V} = 0, \frac{\partial D}{\partial I} = \alpha, \frac{\partial d}{\partial T} = -(u + m + w), \frac{\partial D}{\partial R} = 0$$
$$E = \omega T - (n + \mu) R$$
$$\frac{\partial E}{\partial S} = 0, \frac{\partial E}{\partial V} = 0, \frac{\partial E}{\partial I} = 0, \frac{\partial E}{\partial T} = \omega, \frac{\partial E}{\partial R} = -(n + \mu)$$

The Jacobian matrix is therefore given below

$$J(\sum f) = \begin{pmatrix} -(a+\mu+\beta) & b & 0 & 0 & 0 \\ a & -(b+\mu) & 0 & 0 & 0 \\ \beta & 0 & -(\mu+\alpha) & 0 & 0 \\ 0 & 0 & \alpha & -(\mu+m+\omega) & 0 \\ 0 & 0 & 0 & \omega & -(n+\mu) \end{pmatrix}$$

At disease free equilibrium

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$$(S^*,V^*,I^*,T^*,R^*)=\left(\frac{(b+\mu)\lambda}{(a+\mu)(b+\mu)-ab},\frac{a\lambda}{(b+\mu)(a+\mu)-ab},0,0,0\right)$$

$$J(\sum f)=\begin{pmatrix}-(a+\mu) & b & 0 & 0 & 0\\ a & -(b+\mu) & 0 & 0 & 0\\ 0 & 0 & -(\mu+\alpha) & 0 & 0\\ 0 & 0 & \alpha & -(\mu+m+\omega) & 0\\ 0 & 0 & 0 & \omega & -(n+\mu)\end{pmatrix}$$

the characteristics equation is given :

$$J(\sum f-\lambda I)=\begin{pmatrix}-(a+\mu) & b & 0 & 0 & 0\\ a & -(b+\mu) & 0 & 0 & 0\\ 0 & 0 & -(\mu+\alpha) & 0 & 0\\ 0 & 0 & \alpha & -(\mu+m+\omega) & 0\\ 0 & 0 & 0 & \omega & -(n+\mu)\end{pmatrix}-\begin{pmatrix}\lambda & 0 & 0 & 0 & 0\\ 0 & \lambda & 0 & 0 & 0\\ 0 & 0 & \lambda & 0 & 0\\ 0 & 0 & 0 & \lambda & 0\\ 0 & 0 & 0 & 0 & \lambda\end{pmatrix}$$

$$=\begin{pmatrix}-(a+\mu)-\lambda & b & 0 & 0 & 0\\ a & -(b+\mu)-\lambda & 0 & 0 & 0\\ 0 & 0 & -(\mu+\alpha)-\lambda & 0 & 0\\ 0 & 0 & \alpha & -(\mu+m+\omega)-\lambda & 0\\ 0 & 0 & 0 & \omega & -(n+\mu)-\lambda\end{pmatrix}$$

the determinant gives

$$\{-(a+\mu)-\lambda\}\{-(b+\mu)-\lambda\}\{-(\mu+\alpha)-\lambda\}\{-(u+m+\omega)-\lambda\}\{-(n+\mu)-\lambda\}=0$$

Collinear

$-(a+\mu)-\lambda=0$ or $-(b+\mu)-\lambda=0$ or $-(\mu+\alpha)-\lambda=0$ or $-(n+\mu)-\lambda=0$, or $-(u+m+\omega)-\lambda=0$

$\lambda_1=-(a+\mu)$ or $\lambda_2=-(b+\mu)$ or $\lambda_3=-(\mu+\alpha)$ or $\lambda_4-(n+\mu)$, or $\lambda_5=-(u+m+\omega)$

Since $\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5<0$, then the disease free equilibrium is stable.

Local Stability Analysis of Endemic Equilibrium State

An important criterion by Routh-Hurwitz gives the necessary and sufficient conditions for the all roots of the characteristics (with real coefficients) to lie in the

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left half of the complex plane. In other words, all the roots of the polynomial are negative or have real roots if and only if the determinant of all Hurwitz matrices is positive.

The Jacobean matrix of equations (1-5) at $E_1^{**}(S^{**}, V^{**}, I^{**}, T^{**}\ R^{**})$ as in Joyce et al (2015) is given by

$$\frac{\partial f}{\partial x}(E_1^{**}) = \begin{pmatrix} -(a+\mu+\frac{I^{**}}{N}) & b & -\frac{S^{**}}{N} & 0 & 0 \\ a & -(b+\mu) & 0 & 0 & 0 \\ \frac{I^{**}}{N} & 0 & \frac{S^{**}}{N}-(\mu+\alpha) & 0 & 0 \\ 0 & 0 & \alpha & -(\mu+m+\omega) & 0 \\ 0 & 0 & 0 & \omega & -(n+\mu) \end{pmatrix}$$

$$-\frac{1}{N}\left((n+\mu)\ (\mu+m+\omega)\ (\mu^3 N + (2\alpha N + I^{**} + Nb - S^{**})\ \mu^2 + (b+\alpha)\ (\alpha N + I^{**} - S^{**})\ \mu + I^{**} b\alpha)\right)$$

Hence, by Routh-Hurwitz criteria, the Endemic Equilibrium State is unstable, since

the determinant is negative.

Numerical Simulation

The model variable and parameter values typically must be assessed in view of chickenpox epidemiology. It is often a tedious task to obtain a reliable variable and parameter values for the purpose of simulation and projection of chickenpox in a population. This is because it involves a task of searching for well accepted and reliable data values of variables and parameters. The values of parameters and variables utilized in this research are tabulated in Table 2 with references been made to their sources.

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Table 2: Parameters values of chickenpox dynamics.

S/N	PARAMETERS	VALUES	SOURCES
1	μ	0.03	Ofori 2009
2	b	0.20	Assumed
3	λ	0.05	Ogabi 2009
4	α	0.0000548	Olaniyi 2013
5	n	0.60	Assumed
7	β	0.2	Assumed
8	m	0.51	Assumed
9	ω	0.01	CDC 2004
10	a	0.5	Assumed

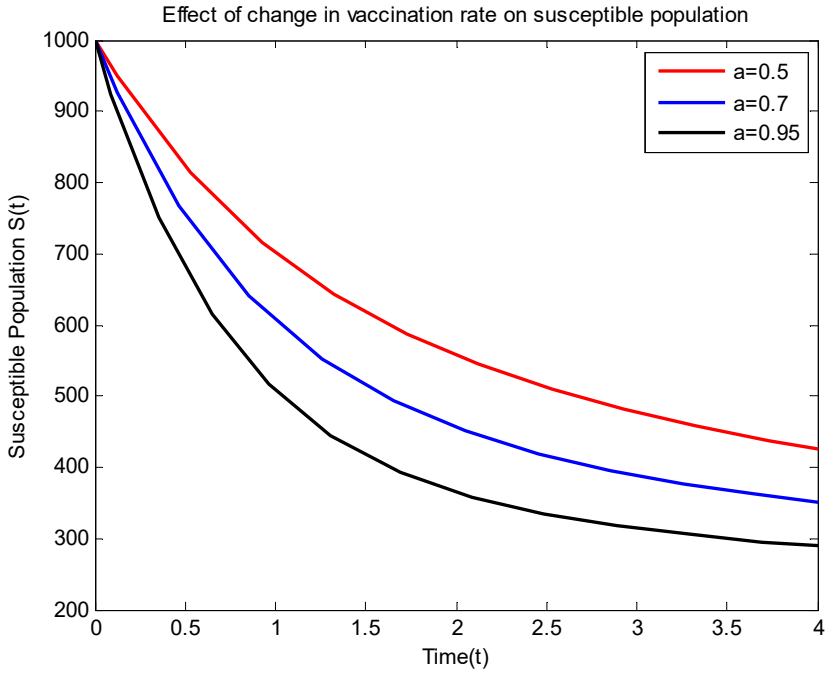

Fig.2is the graph of susceptible individual against time in an outbreak varying the proportion of newborns vaccinated (a=0.5, a= 0.7, a= 0.95).

We observed that as the outbreak occurs, later the vaccination campaign begins to yield result, decreasing the total number of susceptible individuals such a decrease in susceptible individual will automatically bring a decrease in the number of sick individuals and hence control the disease eruption.

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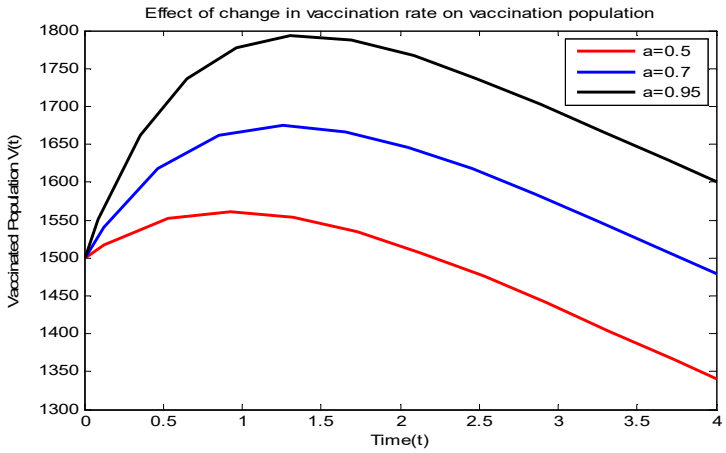

Fig. 3 is the graph of vaccinated individual against time. Varying the proportion of newborns vaccinated (a= 0.5, a=0.7, a= 0.95)

It can be observed from fig (3) that the number of vaccinated individual increases with increase in the vaccination courage of newborns.

It shows that vaccination campaign centered in newborns is the best for the growth of any given population/community, the instant result is as a result of mass vaccination. If vaccine failure is low, then the disease can be eradicated with time.

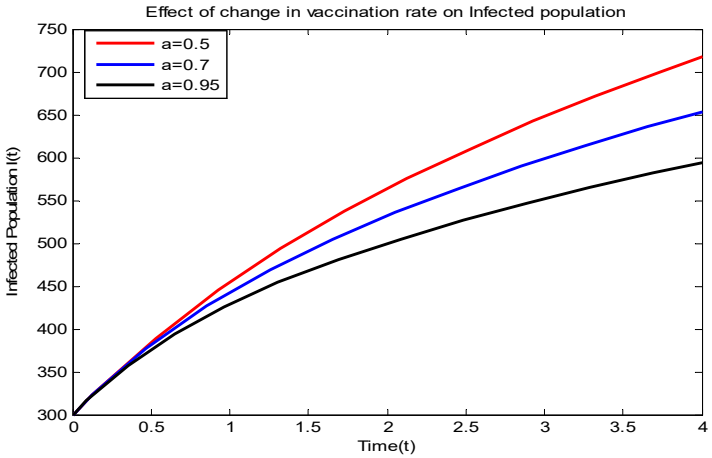

Fig. 4 is the graph of infected population against time in an outbreak, varying the proportion of newborns vaccinated (a= 0.5, a= 0.7, a=0.95)

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It can be seen that as the number of vaccinated individuals increases, the total number of infected individual decreases, but does not yield instant result to fight the disease, this could be as a result of low vaccination of coverage, such a decrease in number infected individuals brings reduction in number of sick individual and hence chickenpox vanishes gradually.

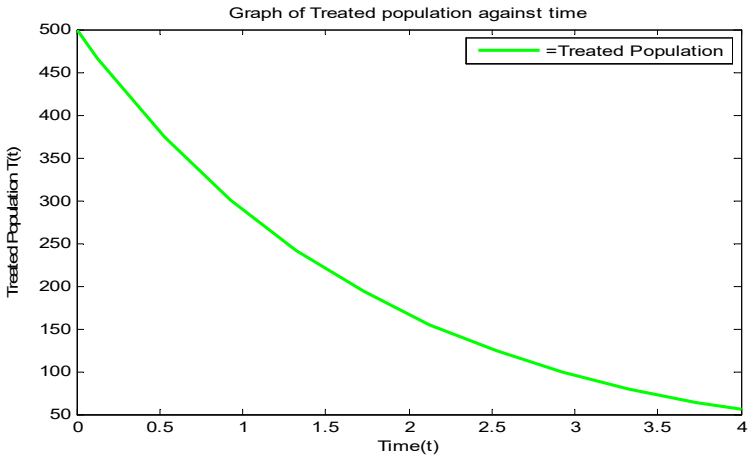

Fig. 5 is the graph of treated individual against time.

It can be observed that the number of treated individuals decline with decreasing time. This declination tends to reduce the number of infected individuals and with this optimal vaccination coverage, then the community can be chickenpox free.

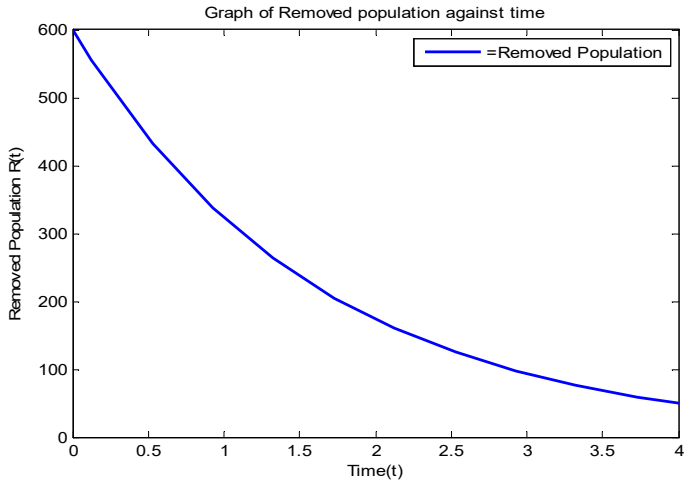

Fig. 6 is the graph of Removed individual against time. It can be seen that the number of removed individuals decreases with time.

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It can also be observed from the graph that when no new borns are vacinnated there is declination in the removed individuals, this might be due to natural immunity of the sick ones.

The rapid declination of the graph above shows that chicken pox can be eradicated from the community.

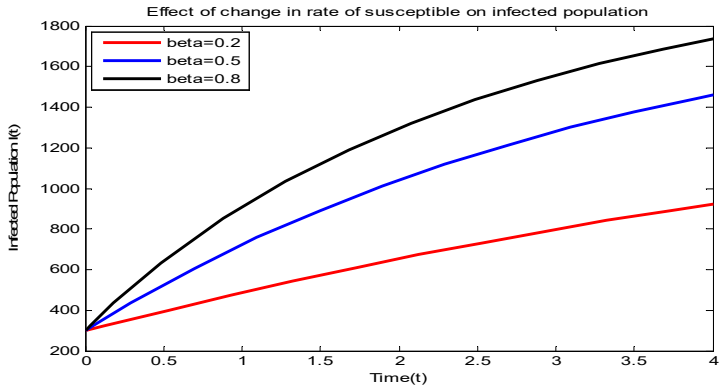

Fig. 7 is the graph of infected individual in an outbreak, varying the proportion of susceptible individuals ($\beta = 0.2, \beta = 0.5, \beta = 0.8$).

It can be seen from the graph above that the number of infected individual increases as the rate of susceptible increases as a result of little or no vaccination.

The graph colored black in figure 7 shows that the maximum infected individuals is approximately 95% when no vaccine is applied. It can be deduced from the graph that without the application of vaccine and treatment, population of the entire community can die off.

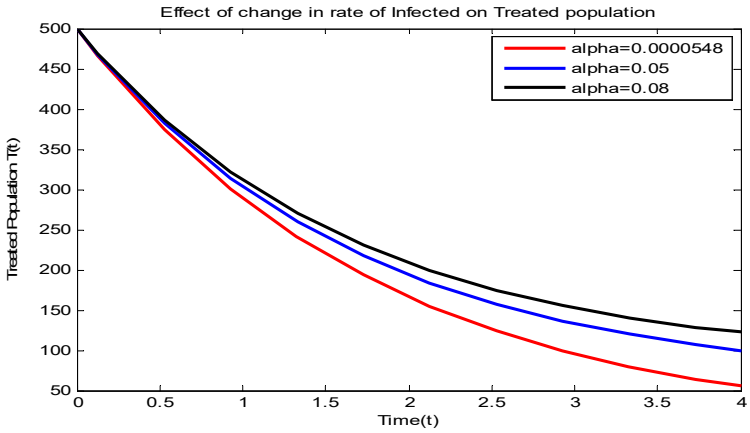

Fig. 8 is the graph of treated population in an outbreak, varying the proportion of infected individuals,($\alpha = 0.0000548, \alpha = 0.05, \alpha = 0.08\alpha$).

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It can be seen from the graphs that the numbers of treated individuals incline as the proportion of infected individual increases i.e. the more Increase in the rate of infected individuals the more increase in treated individuals but insignificant initially, which could be as a result of loss of immunity.

It can be deduced that at high treatment and vaccination infected rate will be deduced.

Conclusion

A mathematical model for the control of chickenpox with vaccination and treatment strategies was developed and analyzed in this study. The model is a first order Ordinary Differential Equations, in which the human population is divided into five compartments namely; Susceptible individuals (S), Vaccinated individuals (V), Infected individuals (I), Treated individuals (T) and Removed individuals (R). The model solution was found to be positive (establishing the fact that human population cannot be negative).The equilibrium states were obtained and their stabilities were analyzed. The result shows that the disease free equilibrium state is stable and the criteria for stability of the endemic equilibrium state are established. This suggests that the combination of vaccination and treatment can used to handle the menace of chicken pox in the population.

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