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FORECASTING NIGERIAN STOCK INDEX BASED ON ASYMMETRY GARCH MODELS

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ABSTRACT

This paper aims at evaluating volatility forecasts of the Nigerian Stock exchange rate obtained through Asymmetric models. We make use of Monthly data from January, 2000 to January, 2012 to evaluate the parameters of each model and produce volatility estimates. The results show that the coefficient of ω (a determinant of the presence of volatility clustering) is statistically significant in the EGARCH model; this appears to show the presence of volatility clustering. The forecasting ability is subsequently assessed using the symmetric lost functions which are the Mean Absolute Error (MAE), Root Mean Absolute Error (RMAE), Mean Absolute Percentage Error (MAPE) and Theil inequality Coefficient. The results show that GJR-GARCH model provides best estimates for persistence, volatility clustering and leverage and asymmetric effects. The results also show that when the leverage and asymmetric effects are absent or minimal, GARCH model provides the most accurate forecast of future volatility.

KEYWORDS: Volatility, GARCH, Asymmetric models, Nigerian Stock Exchange Index

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INTRODUCTION

An exchange rate is the current market price for which one currency can be exchanged for another. For instance, if the Euro exchange rate for the United States Dollar stands at 1.3, this means that 1 euro can be exchanged for 1.3 U.S. dollars. Because exchange rates plays such an important role in a country's competiveness level, currency exchange rates are among the most analyzed and forecasted indicators in the world.

While much of the published work on GARCH appears to be concerned with extending the original specifications in order to capture particular features of financial data such as asymmetry, there are many practical applications of GARCH models and many of these are concerned with forecasting. Even with the growth in popularity of non-linear forecasting models, such as nearest-neighbor non-parametric forecasting and locally weighted regression-forecasting, GARCH models remain a popular forecasting model when dealing with financial time series. Meade (2002) compares forecasts of high- frequency exchange rate data made a linear AR-GARCH model with forecasts made using non-linear techniques and finds that the AR-GARCH forecasts are not improved upon by the non-linear methods he considered. Of course, the results in Meade (2002) paper are specific to a particular data set, and moreover his 'good' forecasting results are for conditional mean forecasts (the forecasts of the series itself are referred to as 'conditional mean forecasts' or 'forecasts of the conditional mean').

Similarly, GARCH models allow forecasts of the conditional variance of a time series in addition to forecasts of the conditional mean to be computed. While computing forecasts of the conditional variance of a time series is straightforward to do using GARCH models (for more detail, see equations 13, 14, 15 and 16) there is still debate on how best to evaluate forecasts of the conditional variance. When evaluating the accuracy of a particular model for computing conditional mean forecasts, it is traditional to estimate the model using a subsample of the full sample of data and then compare the forecasts with the observed future values of the series using a standard measure of forecast accuracy, such as forecast MAE.

However, when forecasting the conditional variance of a time series, such as stock returns, the observed values of the conditional variance of the series are not available for comparison, even if sample observations are held back when estimating the GARCH model. Consequently, it is traditional to use the squared values of the data as a proxy for the observed values of the conditional variance of the series. The forecasts of the conditional variance can then be compared with this proxy and the forecasts can be evaluated in the same way as forecasts of the series itself. The problem with this approach (discussed in detail by Anderson and Bollerslev (1998)) is that the squared values of the series are sometimes a very poor proxy of the conditional variance. As a result, many applied studies have reported that the GARCH model produces poor forecasts of the conditional variance (see, for example, Cumby *et al.* (1993); Jorion (1995); Figlewski (1997)).

Anderson and Bollerslev (1998) focus on evaluating the forecasts of the conditional variance of a returns series using alternative measures to the squared returns as a proxy for observed values of the conditional variance. Their analysis shows that in fact the GARCH model is capable of producing very accurate forecasts of the conditional variance of a time series. We consider forecasting both the conditional mean and conditional variance of our returns series; however, since we are merely illustrating the application of the GARCH model, we take the traditional route of employing the squared values of the S&P Composite returns series as a proxy for the observed conditional variance of the series. This should be borne in mind when interpreting our evaluation measures for the conditional variance forecasts.

Sahoo (2012) examines the volatility factor of the Indian Rupee but also the spillover effects emanating from exchange rates of the Brazilian Real, the Russian Ruble, the South Korean Won, the Singapore Dollar, the Japanese Yen, the Swiss France, the British Pound Sterling and the Euro. Among the class of ARCH and GARCH models, the study employs a two-step multivariate AR(1)-GARCH(1,1) framework to examine the dynamics of exchange rate volatility and its spillovers which is also corroborated by examining simple pair-wise Granger causality tests. We found evidence of conditional autocorrelation and persistence of volatility in daily exchange rates of all nine currencies. The findings also support the view that volatilities observed in the exchange rate of the leading currencies transmit to volatility in the daily exchange rate of the Indian Rupee. However, as the spillover coefficients are smaller than the ARCH and GARCH parameters, the volatility in the exchange rate of the Indian Rupee could also be driven by domestic macroeconomic and global factors.

Yakubu and Bello (2014) used GARCH (1,1), GJR GARCH (1,1), TGARCH (1,1), and TS GARCH (1,1) models to determine the volatility modeling of daily exchange rate between US Dollar and Nigeria Naira from 1st June, 2000 to 26th July, 2011. The results from all the models showed that, volatility is persistent (i.e. exceed 1) indicating GARCH (1, 1), GJR GARCH (1,1), and TS GARCH (1,1) models' variances are not stationary but the variances is stationary for T GARCH (1,1) model. Yakubu and Bello (2014) also reported that, the results from all the asymmetric models rejected the hypothesis of leverage effect. The GJR GARCH (1,1) and TGARCH (1,1) models show the existence of statistically significant asymmetric effect. The TGARCH (1,1) and TS GARCH (1,1) models are found to be the best models because they have maximum likelihood and lower Akaike information criteria (AIC) and Schwarz information criteria (SIC).

Since most of the available time series data are asymmetric in nature, the objective of this research is to identify the asymmetric GARCH model with less volatility and high predictive capacity for Nigerian stock index.

MATERIALS AND METHODS

Data:

The daily exchange rates of the US Dollar against the Euro for the period 4th January, 1999 to 26th April, 2013 are used. These make a total of 3602 observations of the spot price and are converted for the needs of fitting the model to a logarithmic returns series. If the price series is denoted $\{e_t\}$, then the log returns series r_t is such that,

$$r_t = \log\left(\frac{e_t}{e_{t-1}}\right) \qquad 1$$

Where e_t is the Euro-dollar exchange rate at time t and e_{t-1} represent Euro-dollar exchange rate at time $t-1$. The r_t of equation (1) will be used in observing the volatility of the exchange rate between euro and United State Dollar over the period 1999-2013.

GARCH (1, 1) Model:

$$h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1} \qquad 2$$

Where:

β_1 Measures the extent to which a volatility shock today feed through into the next period's volatility. $(\alpha_1 + \beta_1)$ Measures the rate at which this effect lies over time. h_{t-1} is the volatility at day $t-1$.

GARCH Model Extensions:

In most cases, the basic GARCH model provides a reasonably good model for analyzing financial time series and estimating conditional volatility. However, there are some aspects of the model which can be improved so that it can better capture characteristics and dynamics of a particular time series. Since bad news (negative shocks) tends to have a large impact on volatility than good news (positive shocks), hence there is need to talk about the GARCH extensions model and we restricted our analysis to the more popular models of asymmetric volatility, such as EGARCH, TGARCH, IGARCH, TS-GARCH, APARCH, GJR-GARCH, etc.

EGARCH Model:

Nelson (1991) proposed the exponential GARCH (EGARCH) model to allow for leverage effects. The model has the following representation:

$$\log h_t = \alpha_0 + \sum_{i=1}^{p}\alpha_i \frac{|\varepsilon_{t-i}| + \gamma_i\varepsilon_{t-i}}{h_{t-i}^{\frac{1}{2}}} + \sum_{j=1}^{q} b_j \log h_{t-j} \qquad 3$$

Where, γ_i = leverage effect coefficient. (if γ_i >0 it indicates the presence of leverage effect). Note that when ε_{t-i} is positive or there is "good news", the total effect of ε_{t-i} is $(1 + \gamma_i)|\varepsilon_{t-i}|$; in contrast, when ε_{t-i} is negative or there is "bad news" the total effect of ε_{t-i} is $(1 - \gamma_i)|\varepsilon_{t-i}|$. Bad news can have a large impact on volatility, and the value of γ_i would be expected to be positive.

TGARCH Model:

Another GARCH variant that is capable of modeling leverage effects is the Threshold GARCH (TGARCH) model, which has the following form:

$$h_t = \alpha_0 + \sum_{i=1}^{p}\alpha_i\varepsilon_{t-i}^2 + \sum_{i=1}^{p}\gamma_i s_{t-i}\varepsilon_{t-i}^2 + \sum_{j=1}^{q}\beta_j h_{t-j} \qquad 4$$

Where:

$$S_{t-i} = \begin{cases} 1\, if\ \varepsilon_{t-i} < 0 \\ 0\, if\ \varepsilon_{t-i} \geq 0 \end{cases}$$

γ_i = leverage effects coefficient. (if $\gamma_i > 0$ it indicates the presence of leverage effect). That is depending on whether ε_{t-i} is above or below the threshold value of zero, ε_{t-i}^2 has different effects on conditional variance h_t: when ε_{t-i} is positive, the total effects are given by $\alpha_i \varepsilon_{t-i}^2$ and when ε_{t-i} is negative, the total effects are given by $(\alpha_i + \gamma_i)\varepsilon_{t-i}^2$. So one would expect γ_i to be positive for bad news to have larger impacts. This model also known as the GJR model, Ding (1993) proposed essentially the same model.

GJR-GARCH Model:

The GJR-GARCH model is another volatility model that allows asymmetric effects. This was introduced by Figlewski (1997). The general specification of this model is of the form:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \left(\alpha_i \varepsilon_{t-1}^2 + \gamma_i s_{t-i}^- \varepsilon_{t-1}^2 \right) + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \qquad 5$$

Where: s_{t-i}^- is a dummy variable which takes the value of 1 when γ_i is negative and 0 when γ_i is positive. In this GJR-GARCH model, it is supposed that the impact of ε_t^2 on the conditional variance σ_t^2 differs when ε_t^2 is positive or negative. A nice aspect of the GJR-GARCH model is that it is easy to test the null hypothesis of no leverage effects. In fact, $\gamma_1 = \ldots = \gamma_q = 0$ means that the news impact curve is symmetric, i.e. past negative shocks have the same impact on today's volatility as positive shocks.

Calculating the *optimal h-step ahead* forecast

It important to note that to calculate the *optimal h-step ahead* forecast of y_t, the forecast function obtained by taking the conditional expectation of y_{T+h} (where T is the sample size) is used. So, for example, in the case of the AR (1) model:

$$y_t = \delta + \rho y_{t-1} + \varepsilon_t \qquad 7$$

where $\varepsilon \sim IID(0, \sigma^2)$, the optimal *h*-step ahead forecast is:

$$E\left(y_{T+h} \middle| \Omega_T \right) = \delta + \rho y_{T+h-1} \qquad 8$$

Where Ω_T is the relevant information set. Therefore the optimal one-step ahead forecast of y_t is simply $\delta + \rho y_{t-1}$. While the forecasting function for the conditional variances of ARCH and GARCH models are less well documented than the forecast function for conventional ARIMA models (see Granger and Newbold (1986), chapter 5, for detailed information on the latter), the methodology used to obtain the optimal forecast of the condition variance of a time series from a GARCH model is the same as that used to obtain the optimal forecast of the conditional mean. Further details on the forecast function for the conditional variance of a GARCH (p,q) process is given below:

Consider the equation for the conditional variance in a GARCH (p,q) model:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \qquad 9$$

Taking conditional expectations and assuming a sample size of T and for convenience that the parameters in the forecast functions are known, the forecast function for the optimal h-step ahead forecast of the conditional variance can be written:

$$E\left(h_{T+h|\Omega_T}\right) = \alpha_0 + \sum_{i=1}^{q} \alpha_i E\left(u_{T+h-i}^2 \middle| \Omega_T\right) + \sum_{j=1}^{p} \beta_j E\left(h_{T+h-j|\Omega_T}\right) \qquad 10$$

Where Ω_T is the relevant information set. $E\left(u_{T+i}^2 \middle| \Omega_T\right) = E\left(h_{T+i} \middle| \Omega_T\right)$ for $i > 0$, $E\left(u_{T+i}^2 \middle| \Omega_T\right) = u_{T+i}^2$ for $i \le 0$ and $E\left(h_{T+i} \middle| \Omega_T\right) = h_{T+i}$ for $i > 1$. $E\left(h_{T+i} \middle| \Omega_T\right)$ is computed recursively. Thus the one-step ahead forecast of h_T is given by:

$$E\left(h_{T+i} \middle| \Omega_T\right) = \alpha_0 + \alpha_1 u_T^2 + \beta_1 h_T \qquad 11$$

The forecast of the conditional variance for GARCH-M models can be obtained in a similar way. Clearly, the forecast functions for some of the extensions of the original GARCH specifications will more difficult to drive. For example, in the GJR-GARCH model recall that the conditional variance is given by:

$$h_t = \alpha_0 + \alpha_1 u_{t-1}^2 + \gamma_1 u_{t-1}^2 I_{t-1} + \beta_1 h_{t-1} \qquad 12$$

Where $I_{t-1} = 1$ if $u_{t-1} > 0$ and $I_{t-1} = 0$ otherwise. Unless $\gamma_1 = 0$, forecasts of the indicator function I_{t-1} need to be computed. The sign of u_{t-1} and therefore the forecasts of I_{t-1} will depend on the assumed distribution for ε_t.

Volatility Forecasts Comparison:

Volatility forecasts comparison was conducted for one-step ahead horizon in terms of Root Mean Square Error (RMSE), Mean Square Error (MSE), Mean Absolute Error (MAE), Mean Percentage Error (MPE), Mean Error (ME), Mean Logarithm of Absolute Error (MLAE) and Theil Inequality Coefficient.

In order to estimate the forecasting performance of some models or to compare several models, we should define error functions. The following are the most used error functions:

1. Root Mean Square Error (RMSE) = $\sqrt{\frac{1}{N}\sum_{t=1}^{N}\left(\hat{\sigma}_t - \sigma_t\right)^2}$ 13

2. Mean Absolute Error (MAE) = $\frac{1}{N}\sum_{t=1}^{N}\left|\hat{\sigma}_t - \sigma_t\right|$ 14

3. Mean Absolute Percent Error (MAPE) = $\frac{1}{N}\sum_{t=1}^{N}\frac{\left|\hat{\sigma}_t - \sigma_t\right|}{\sigma_t}$ 15

4. Theil Inequality Coefficient = $\dfrac{\sqrt{\dfrac{1}{N}\sum_{t=1}^{N}(\hat{\sigma}_t-\sigma_t)^2}}{\sqrt{\dfrac{1}{N}\sum_{t=1}^{N}\hat{\sigma}_t^2}+\sqrt{\dfrac{1}{N}\sum_{t=1}^{N}\sigma_t^2}}$ 16

Where N is the number of out-of-sample observations, σ_t is the actual volatility at forecasting period *t* measured as the square daily return, and $\hat{\sigma}_t$ is the forecast volatility at *t.*

Note that the first three forecast error statistics depend on the scale of the dependent variable. These should be used as relative measures to compare forecasts for the same series across different models; the smaller the error, the better the forecasting ability of that model according to that criterion. The inequality coefficient always lies between zero and one, where zero indicates a perfect fit. Yakubu *et al* (2014)

Note also that the mean squared forecast error can be decomposed as:

$$\frac{1}{N}\sum_{t=1}^{N}\left(\hat{\sigma}_t-\sigma_t\right)^2=\left(\left(\frac{1}{N}\sum_{t=1}^{N}\hat{\sigma}_t\right)-\bar{y}\right)^2+\left(\delta_{\hat{y}}-\delta_y\right)^2+2\left(1-r\right)\delta_{\hat{y}}\delta_y \qquad 17$$

Where $\frac{1}{N}\sum_{t=1}^{N}\hat{\sigma}_t,\bar{y},\delta_{\hat{y}},\delta_y$ are the means and (biased) standard deviations of $\hat{\sigma}_t$ and σ_t, and r is the correlation between $\hat{\sigma}$ and σ. The proportions are defined as:

Bias proportion = $\dfrac{\left(\dfrac{1}{N}\sum_{t=1}^{N}\hat{\sigma}_t-\bar{y}\right)^2}{\dfrac{1}{N}\sum_{t=1}^{N}\left(\hat{\sigma}_t-\sigma_t\right)^2}$ 18

Variance proportion = $\dfrac{\left(\delta_{\hat{y}}-\delta_y\right)^2}{\dfrac{1}{N}\sum_{t=1}^{N}\left(\hat{\sigma}_t-\sigma_t\right)^2}$ 19

Covariance proportion = $\dfrac{2\left(1-r\right)\delta_{\hat{y}}\delta_y}{\dfrac{1}{N}\sum_{t=1}^{N}\left(\hat{\sigma}_t-\sigma_t\right)^2}$ 20

• The bias proportion tells us how far the mean of the forecast is from the mean of the actual series.
• The variance proportion tells us how far the variation of the forecast is from the variation of the actual series.
• The covariance proportion measures the remaining unsystematic forecasting errors.

Note that the bias, variance, and covariance proportions add up to one.
If your forecast is "good", the bias and variance proportions should be small so that most of the bias should be concentrated on the covariance proportions.

Empirical Analysis

Data Properties:
Figure 1 indicates that the series contains trend components. To remove the trend components, we take the first difference (d) of the logarithms (I) of the data and the series are preferred in analysis of financial time series because they have attractive statistical property which is stationary as shown in figure 2.

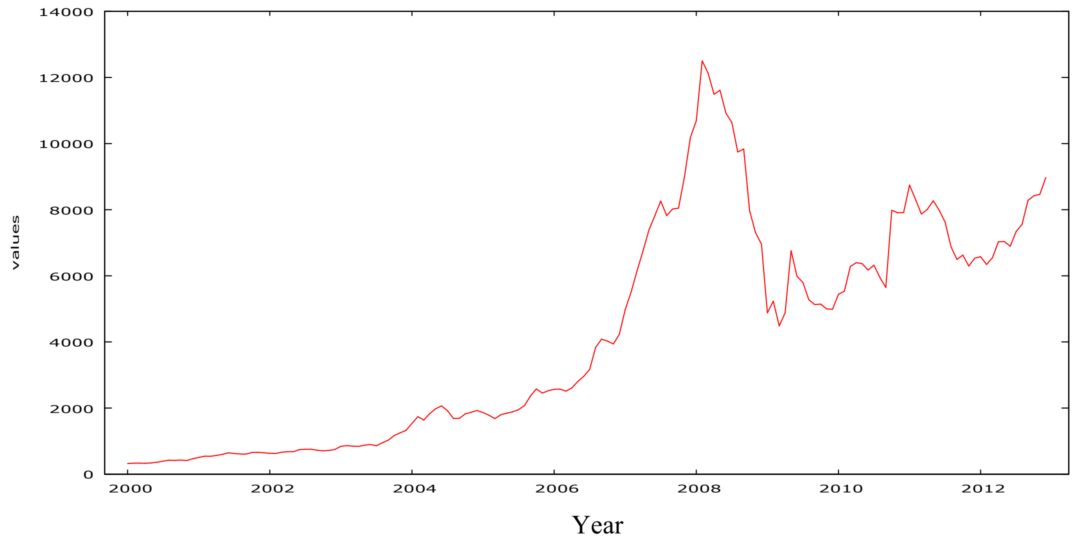

Fig. 1: Nigeria Stock Index (2000-2012)

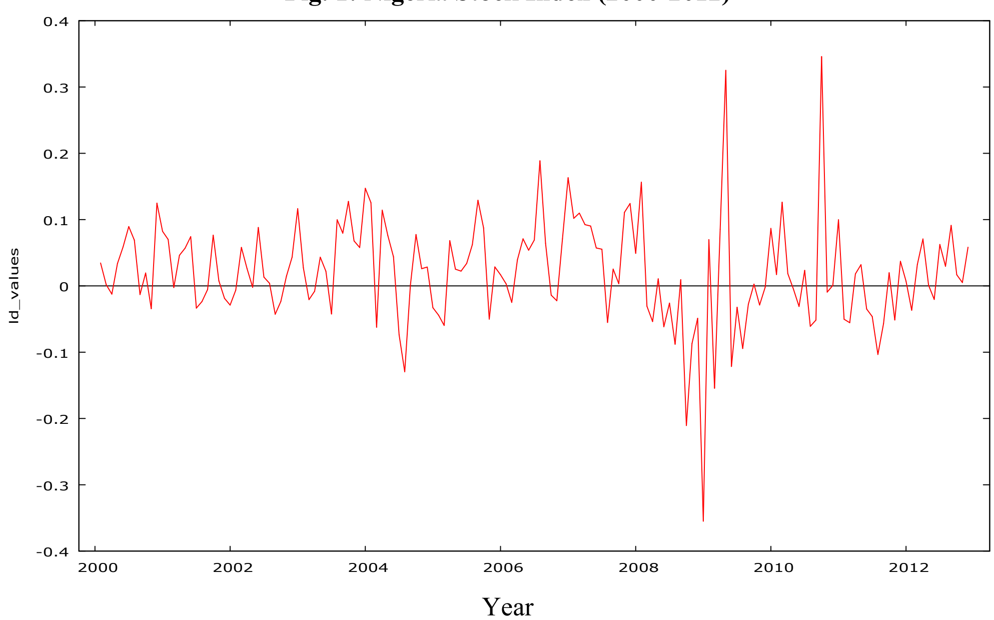

Fig. 2: First Difference of Logarithm for Nigerian Stock Index

Analysis of Some Selected Volatility Models:
The particular optimization routine of the log-likelihood function specified shows that, among ARCH(p)/GARCH(p, q) classes for p [1,5] and q [1,2], the optimal model for the returns is GARCH(1, 1) model. Table 1 gives the parameter estimates of the GARCH (1, 1) and Asymmetric models.

Table 1: Parameter Estimates of GARCH Models for the Periods: January, 2000-January, 2012

Parameter	GARCH (1, 1)	GJR-GARCH(1, 1)	TGARCH(1,1)	EGARCH (1, 1)
Ω	0.000781348	0.00129767	0.000608016	-1.11103
	(0.1014)	(0.2194)	(0.4172)	(0.0017)
α	0.315234	0.151238	0.258367	0.434385
	(0.0773)	(0.8203)	(0.0724)	(0.0080)
γ	-	0.722482		-0.162968
		(0.8311)		(0.0727)
β	0.599505	0.558788	0.704589	0.853034
	(0.0000)	(0.0000)	(0.004)	(0.0000)
AIC	-362.53969	-369.01079	-363.40694	-366.72605
BIC	-350.36599	-353.79366	-351.23324	-351.50892
LL	185.26985	189.50539	185.70347	188.36302

From Table 1, we can deduce the following:

The coefficient of ω (a determinant of volatility clustering) is statistically significant in EGARCH only at 5% level of significance. This appears to show the presence of volatility clustering in the statistically significant model (EGARCH). This means that, the conditional volatility for these models tend to rise (fall) when the absolute value of the standardized residuals is large (smaller).

The coefficient of β (a determinant of degree of persistence) is statistically significant in all the models.. Indeed, the Nigeria Stock exchange rates are characterized by high volatility persistence.

The coefficient of γ (a determinant of asymmetry and leverage effects) is not statistically significant in all models. Henceforth, the GJR-GARCH and EGARCH models are found to be the best models because they have maximum likelihood, lower AIC and lower BIC.

Forecast Evaluation:
We investigate the forecasting performance of all models using 156 observations of the Nigerian Stock Index for the periods stated above.

Table 2: Comparison of the Accuracy Of Volatility Forecasts

	GARCH	GJR-GARCH	TGARCH	EGARCH
RMSE	0.041654	0.051354	0.050051	0.051342
MAE	0.057208	0.043208	0.060208	0.045101
MPE	97.2100	100.0000	97.7700	98.0000
Theil-IC	1.000000	1.000000	1.000000	1.000000
Var Prop	0.020413	0.010413	0.010413	0.010423
Bias Prop	0.799530	0.989587	0.911587	0.989577
Cov Prop	0.180057	0.000000	0.078000	0.000000

Where **RMSE** = Root Mean Square Error, **MSE** = Mean Square Error, **MAE** = Mean Absolute Error, **MPE** = Mean Percentage Error, **ME** = Mean Error, **MLAE** = Mean Logarithm of Absolute Error () and, **Theil-IC** = Theil Inequality Coefficient. **Var Prop** = variance proportion, **Bias Prop** = bias proportion, **Cov Prop** = covariance proportion

From Table 2, we deduce the following;
The GARCH(1,1) has the least RMSE and MPE following by TGARCH(1,1). This implies that GARCH(1,1) is more accurate in volatility forecast when the effects of other stylize empirical facts are minimum. The TGARCH also has the least variance and highest covariance. This implies that is more accurate when correlation between the present and past volatility is high.

Table 3: Comparison with previous research works

	Asymmetric models		**ARIMA(3,1,1)**	**SES**	**ANN**
	GJR-GARCH(1,1)	**EGARCH(1,1)**	**Adebayo *et al.* (2014)**	**Alhassan *et al.* 2014**	**Alhassan *et al.* 2014**
AIC	-369.01	-366.72	1.57	*	*
SIC	*	*	1.86	*	*
HQS	*	*	1.62	*	*
RMSE	0.05	0.05	0.75	2.74	0.07
MAE	0.04	0.05	0.62	0.01	0.00

*Not available

Using RMSE from Table 3, the asymmetric models perform better than ARIMA, SES and ANN models.

CONCLUSION AND RECOMMENDATION

From the results of the analyses, GJR-GARCH provides the better estimate for Nigerian Stock Exchange index especially when the leverage and asymmetric effects are high. This model will capture all the necessary stylize empirical facts (common features) of financial data such as persistence, volatility clustering and asymmetric effects. The results also shows that in the absence of the above effects, GARCH and TGARCH models are the appropriate models for predicting future index for Nigerian Stock Exchange.

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