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APPLICATION OF LOGISTIC GROWTH MODEL TO ENROLMENT DATA

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ABSTRACT

A study on graduate enrolment over a given period of time putting into consideration available infrastructures and manpower was carried out. A modified Logistic growth model was evolved and its application to a data shows that it fits well in periodic enrolment with a target maximum population.

KEYWORDS: Enrolment, Modeling, Logistic Growth, Population, Variable Separable.

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INTRODUCTION

In theory any figure drawn on paper is a tangible aid to the understanding of models. Earliest use of the word model denoted a set of architects plan. However, models could be defined as the representation or abstraction of an object or situation. It shows the relationship and the interrelationship of actions and reactions in terms of cause and effect. Models are less complex than the reality they are employed to explain and describe some aspect of the system under study as well as provide a stepping stone on experimental investigation of the reality itself with a high degree of accuracy at less cost. Though large numbers of variable may be required to study a situation with reasonable accuracy, a small number of variables usually account for most characteristics of the situations, hence to find the right variable and the correct relationship between them is the concern of model formulation. The work on growth model using the tool of differential equations as reported in Boyce and Diprima (1992) started with Verhulst (1838), who concerned himself with the mathematical formulation of predicting the human population of some countries and one of the equations he studied was a non linear Bernoulli's differential equation of the form $\frac{dy}{dx} + p(x)y = q(x)y^n$, $n \neq 0$ or 1 but was faced with the difficulty of not having population data to verify his model at that time. In 1840 as reported in Boyce and Diprima (1992), Verhulst simplified the first order non-linear ordinary differential equation in the form $\frac{dp}{dt} = p(a - bp)$, where a and b are positive constants and p is the size of the population. He called the equation Logistic law and later introduced it into population dynamics. In Boyce and Diprima (1992), Lotka and Volterra(1925) also formulated the unrestricted simple generalized model for population growth that predicted only ever increasing population. Boyce (1981) studied the restricted growth of population for investigating the limitations set by physical and other factors. The model was also extended to investigate some experimental data in agriculture. Analysis has it that it fits some quite well. Oruh and Onwanta (2005) studied the limiting population model for single species of organism in an ecological environment. The suiting models were verified using data obtained from the growth of yeast in a culture. Fan *et al*., (2001) examined the global stability of an epidemic model with recruitment and a varying total population size. Omosigbo (2000) developed unlimited growth model to examine the implication of the National University Commission of Nigeria policy on Students' enrolment. The model is given by

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$p(t) = p(t-1) + I(0)[1-\exp(-\alpha k)]\exp[\alpha(t-1)]$, where p(t) is the total population of students at epoch t, I(0) is the initial population of the intake, α is the growth rate, t is the epoch in time and k is the duration of the degree programme at epoch t. In all these studies, little attention was given to enrolment that has a target maximum for the attainment of stability. Ekhosuchi and Osagiede (2008) in the review of some studies carried out reported that, the models are mathematically and statistically sophisticated and that the methods discussed in Ugwuowo and McClean (2000) as reported by them can serve as reference tools in modeling heterogeneity in wastage and enrolment projection for the educational system of a given country. Our aim is to study in particular the enrolment of qualified persons in an establishment that has a maximum number it can accommodate over a given period with a view to guiding against overcrowding.

Mathematical Formulation of the Physical Problem

The Logistic law with the inclusion of boundary conditions which set a target for growth with an upper bound (M) at time (t), modified the law for our study and it takes the form

$$\frac{dx(t)}{dt} = kxm = -kx^2 \qquad k > 0 \tag{1}$$

$$x(0) = x_0, \quad x(t_\alpha) = x_\alpha \tag{2}$$

where x(t) is the size of the enrolment under investigation and k is a growth factor.

Method Solution

Equation (1) is a non linear differential equation, it is linearized and applying variable separable technique, the solution takes the form

$$x = \frac{MA\exp Mkt}{1 + A\exp Mkt} \tag{3}$$

where A = expMc and c is constant of integration.

Imposing the boundary conditions of equation (2) in equation (3) results in

$$A = \frac{x_0}{M - x_0}$$

and

$$k = \frac{1}{Mt_\alpha}\ln\left(\frac{x_\alpha}{q}\right)$$

where $q = M\left(\frac{x_0}{M-x_0}\right) - x_\alpha\left(\frac{x_0}{M-x_0}\right)$

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The complete solution of the modified Logistic law is therefore

$$x = \frac{M\left(\frac{x_0}{M - x_0}\right)\exp\left[\frac{1}{t_\alpha}\ln\left(\frac{x_\alpha}{q}\right)\right]t}{1 + \left(\frac{x_0}{M - x_0}\right)\exp\left[\frac{1}{t_\alpha}\ln\left(\frac{x_\alpha}{q}\right)\right]t}$$

Differentiating equation (1) and at maximum point, we get

$$x = \frac{M}{2} \qquad (5)$$

The implication of equation (5) is that, the maximum point is at half of the maximum population size. Thus, when enrolment reaches half of the maximum size (M), the enrolment rate is most rapid, and then starts to increase slowly towards M.

Model Verification

To verify the model, we consider the data below representing the intake of students of a private tertiary institution in Nigeria from 2002-2006

Table 1: Enrolment and computed number of students for different years

Time (t) in years	Enrolment (x)	Enrolment(x) from model
2002 (0)	421	421
2003 (1)	620	706
2004 (2)	664	812
2005 (3)	728	836
2006 (4)	840	840

The value of M is now used together with the boundary conditions from the data x(0) =421 and x(4) = 840 in equation (4) results in

$$x = \frac{842\exp(1.65)t}{1 + \exp(1.65)t} \qquad (6)$$

Using equation (6) we generate column 3 of Table 1

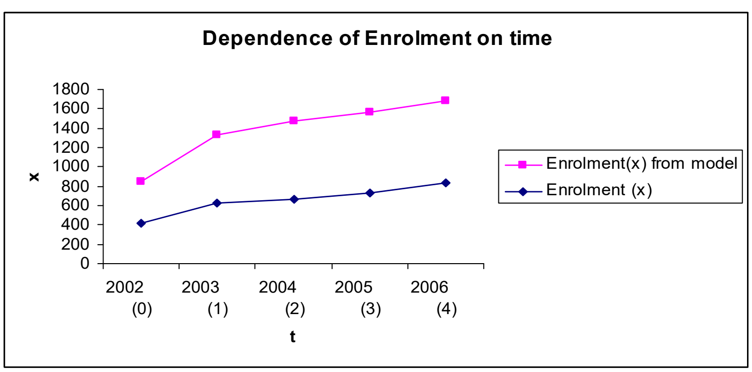

Figure 1: Dependence of Population size on time interval.

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DISCUSSION

The modified Logistic law has been tested with a data. Figure 1 and Table 1 showed that in any situation where a maximum is set over a given period for enrolment, the modified law is effective in proportionately spreading the intake over the time interval. As the curves in figure 1 approaches M so also the t approaches the target specified time for stability to be attained. When this is achieved, the rate of intake equals the rate of passing out. If $1 << t < \infty$, then equation (6) reduces to x = M, the stability point of the model.

Assumptions

For effective use of the model, the following assumptions at imperative at stability;
If it is for students' intake, no student repeats a particular class and all classes have equal number of students.
If it is for establishments, the number of persons employed is equal to that retired and all departments have equal number of staff.

CONCLUSION

The authors are of the view that there is no best model, each model is appropriate to a given circumstance, however the modified Logistic law proposed and tested with a given data is recommended for use particularly when the target maximum is projected for stability to be attained.

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