Continental J. Applied Sciences
Ojemeri et al. (2025) 20 (1): 61 – 79 Doi: 10.5281/zenodo.14925803

Research Article

Dynamics of Viscous Casson Fluid on Combined Convection Flow with Exothermic Chemical Reaction in an Upstanding Tube

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Abstract

The focus of this research is the investigation of steady state fully developed mixed convection flow in a chemically reacting fluid (exothermic fluid) in the presence of Casson fluid across a vertical tube. Subject to suitable boundary conditions, the temperature and velocity equations are solved in non-dimensional form employing regular perturbation approach. The basic flow patterns of temperature and velocity flow are explored as an implication of controlling characteristics such as chemical reaction parameter, Casson fluid parameter, mixed convection parameter and constant pressure gradient. The findings are carefully investigated and graphically represented with the help of illustrative graphs. It was observed that raising the value of the positive mixed convection parameter enhances the fluid velocity while the negative mixed convection parameter suppresses the fluid flow. Additionally, increasing the values of chemical reaction parameter is noticed to improve the fluid temperature and velocity respectively, whereas increasing Casson parameter tends to suppress the fluid velocity.

Keywords: Casson fluid, Combined (Mixed) convection, Exothermic reaction, Vertical tube

ISSN: 3043–6087. Science and Education Development Inst., Nigeria

Received: 28 November 2024 **Accepted: 24 February 2025**

Citation: Ojemeri, G., Shuiabu, A., Abubakar, A.Y., Usman, I.O., and Ahmad, I.S. (2025). Dynamics of Viscous Casson Fluid on Combined Convection Flow with Exothermic Chemical Reaction in an Upstanding Tube. Continental J. Applied Science, 20(1), 61–79. Doi: 10.5281/zenodo.14925803

List of Symbols
g=gravitational acceleration
b= channel width
λ= Chemical reactant parameter
Gre = Mixed convection parameter
Γ= Constant pressure gradient
ξ= Casson fluid parameter
T=temperature of the fluid
T_0=reference temperature
u=velocity component in x direction
U=dimensionless velocity

Greek letters
β =thermal expansion coefficient
$\beta_t \beta_v$ =dimensionless variables
μ =dynamic viscosity
α =thermal diffusivity
γ_s = ratios of specific heats ($C_p C_v$)
σ =electrical conductivity of the fluid
ρ = density
v=fluid kinematic viscosity

Introduction

The term "Casson fluid" describes a type of non-Newtonian fluid characterized by variable viscosity. The dominance of viscous force in this system can be attributed to the action of variable viscosity in the fluid. Fluids commonly employed in many applications include paints, diverse polymer solutions, blood, honey, etc. Typical instances of Casson fluids encompass synthetic lubricants, mud extraction, clay coatings, and biomedical fluids. The Casson fluid simulations that are readily accessible are classified based on their distinct rheological properties, including Oldroyd-B, Eyring-Powell, Oldroyd-A, Maxwell, Carreau, Jeffrey, and Burger. In the field of modern technology, there are some flow features that defy comprehension when analyzed solely through the lens of the

Newtonian flow model. Consequently, the utilization of the non-Newtonian fluid notion is more advantageous. Some noteworthy works like Amaraikannan *et al.* (2021a), Shahzad *et al.* (2021), Amaraikannan *et al.* (2021b), and Ali *et al.* (2021) provide valuable insights for readers on this topic. Several works have been published to demonstrate the effects of Casson fluid parameter through various geometric shapes under different settings. To this end, Hamza *et al*. (2024) recently studied the effect of Casson fluid with heat absorption affected by symmetric wall temperature and concentration conditions. Onwubuya *et al.* (2024) recently simulated the impact of magnetohydrodynamics (MHD) Casson fluid affected by porosity and thermal radiation factors along a permeable channel in an oscillating system. Ojemeri and Abubakar (2024) investigated the impact of Casson fluid on magnetized oscillatory flow along a permeable plate immersed in porous medium is investigated in the optically thin thermal radiation regime. Ojemeri *et al*. (2023c) put forth magnetized Casson flow of an incompressible fluid with thermal radiation impact in an up-facing porous plate. Also, Nandeppanavar (2018) described the movement pattern of Casson fluid through a moving plate. Ahmad *et al*. (2019) introduced a novel approach to modelling a Casson fluid with fractional derivatives in a more recent study. Further, Sarkar and Endalew (2019), Hamid *et al.* (2019), Das *et al.* (2021), Amjad *et al.* (2021), and Sarkar *et al.* (2020) have published the flow behavior of the Casson fluid in various physical settings.

The buoyancy force created from different temperature regions and lid-driven force from the walls -movement taking place in the same system can be acknowledged as mixed convection. Mixed convection is interrelated with Richardson number, Ri, which originates from Grashof number and Reynolds number. Many studies agreed that altering Ri can modify the types of convection in investigated and related factors related to various cases such as home ventilation, fire prevention, solar collector, drying technologies, and chemical processing, as indicated by (Saedodin et al. 2013). Due to its emerging relevance in numerous realistic systems of immense applications in thermal energy storage, geothermal energy utilization, recoverable systems, petroleum reservoirs, metallurgy, polymer processing, geophysics, and other fields, the modelling of mixed convection flow on stretching surfaces has piqued the interest and focus of researchers (Mandal et al. 2020). Mixed convection can be used for the dispersion of chemical contaminants in various processes in chemical industry and in transporting of heated or cooled fluids and so on, as reported by Lavine (1988). Much emphasis has been paid to the investigation of mixed convective flow along a rigid plate lately because it is essential for a variety of applications, including the electronic cooling devices by fans, the cooling of nuclear reactors during an unexpected failure, the placement of heating systems in minimal-velocity environments, solar cells, and others. (Aydin and Kaya, 2013). In the last decade, a number of studies have been conducted in this field. In this context, Ahmad *et al.* (2023) evaluated the effects of a pressure gradient that was exponentially increasing or decreasing on the transient mixed convection flow of viscous reactive fluids in a vertical tube between two concentric tubes with r = 0 and r = b. The nonlinear partial differential equation governing the flow formation are solved using the implicit finite difference form where all time differentials were calculated using the forward difference formula. Hamza *et al.* (2023c) analytically and numerically examined the implication of time-dependent hydromagnetic flow of an exothermic fluid in a vertical channel regulated by convective heating and Navier slip conditions using homotopy perturbation method and implicit finite difference scheme. Hindebu *et al.* (2022) analyzed the hydrodynamic and heat transfer problems of unsteady mixed convection flow of nanofluid in a micro-channel filled with porous material. Ajibade and Umar (2021) scrutinized the effects of viscous dissipation on a steady mixed convection Couette flow of heat generating/absorbing fluid. In a vertical porous annulus, Hamza *et al*. (2019) discussed the combined impacts of the Soret effect, mixed convection, and radial magnetic field flow of viscous reactive fluid. Jafar *et al.* (2022) reported that the buoyant force has a substantial impact on the flow field for vertical or inclined surfaces. Daniel and Daniel (2015) presented the MHD mixed convection flow having radiation effect towards a stretching porous surface by utilizing the homotopy analysis method (HAM) while Jamaludin *et al.* (2020) examined the influence of heat emission/absorption in MHD mixed convection stagnation point in a hybrid nanofluid. They reported that the heat transfer of the conventional alumina/water nanofluid is greater than the copper–alumina water nanofluid with the increment in the heat source and sink parameter. Hayat *et al.* (2016) investigated mixed convection flow of viscoelastic nanofluid by a cylinder with variable thermal conductivity and heat source/sink. Hamza and Abdulsalam (2022) investigated the influence of chemical kinetic exponent on transient mixed convective hydromagnetic flow in a vertical channel with convective boundary condition using homotopy perturbation method.

The simulation of flow models through viscous or chemically reactive fluids with MHD free convection contemplated by different geometrical surfaces has gained growing attention over the years attributable to its possible applications in many chemical engineering and petroleum chemical industries. The interaction between chemical reaction and natural convection occurs widely in chemical engineering applications, for instance, tubular laboratory reactors, chemical vapour deposition systems, the oxidation of solid materials in large containers, in transpiration cooling of re-entry vehicles and rocket boosters, film vaporization in combustion chambers, and cross-hatching on ablative surfaces, the synthesis of ceramic materials by a self-propagating reaction, combustion in underground reservoirs for enhanced oil recovery and reduction in hazardous combustion product by using catalytic porous beds and many others (Muthura and Srinivas 2010). The modelling of reactive viscous fluids was first presented by Frank–Kamenetskii (1969). According to Makinde (2008), most lubricants used in engineering and industrial processes, such as synthetic esters, hydrocarbon oils, polyglycols, and so on, are reactive. The analysis of chemical reaction of an MHD fluid flows have been published by many authors through various flow channels. With these concerns in mind, Ahmad and Jha (2015) conducted a numerical and an analytical investigations of the influence of heat transfer flow of an exothermic chemical reacting fluids in an upstanding porous pipe. Their findings depict that heat transmission amount on pipe surfaces, whether transient or steady state, increases with time. Ahmad *et al*. (2017) extended their previous work by investigating the steady state mixed convective heat and mass transfer flow of an exothermic chemically reacting fluid in an upward porous conduit affected by Soret effect. According to their findings, the Soret effect and mixed convection had a remarkable impact on the fluid velocity in the pipe. Obalulu *et al*. (2021) discussed the consequences of variable electrical conductivity on non-Newtonian fluid flow restricted to two upstanding plates in the coexistence of Arrhenius energy and exothermic chemical reaction. Hamza *et al.* (2023a, 2023b) discussed the consequences of Arrhenius-controlled chemical reaction and Hall current on hydromagnetic free convection flow of an incompressible fluid along an upstanding microchannel subjected to appropriate boundary conditions using homotopy perturbation method. Ojemeri *et al.* (2023a) focused their search on the analytical treatment of Arrhenius-controlled fluid in an upright microchannel saturated with porous material using homotopy perturbation method (HPM) restricted to an appropriate boundary conditions. Their results showed that the variations of chemical reaction, Darcy number, rarefaction and wall-ambient temperature parameters substantially dictate the fluid flow and volume flow rate respectively. Ojemeri *et al.* (2023b) explored the analytical solutions of natural convection of a chemical reacting fluid in the involvement of Soret and radial magnetic field in an annular upstanding permeable zone within concentric cylinders. It is noteworthy to report that raising the levels of Frank–Kamenetskii, sustentation, and thermo-diffusion parameters increases fluid velocity whereas reducing the radial magnetic field effect weakens the fluid flow. Additionally, it is significant to report that the sheer stress on the annular walls can be effectively regulated by applying appropriate values of magnetic number. In another paper, Ojemeri and Hamza (2022) put forth a computational treatment of an Arrhenius kinetically propelled heat emission and absorption fluid in a microchannel. Jha *et al*. (2011a, 2011b) studied the time-dependent natural convection of an exothermic reaction fluid in an upright channel restricted to two immeasurable upright parallel plates as well as in a tube. They concluded that the amount of heat transmission and sheer stress on both walls increases as the chemical reactant parameter is increased. References like Ali *et al*. (2014) and Hamza (2016) shed more light on this phenomenon.

This aim of this current research is to modify the work of Ahmad *et al.* (2023) by incorporating and investigating the impact of Casson fluid on steady mixed convection flow of chemical reacting fluid in a vertical channel using regular perturbation approach. Researches on the effects of Casson fluid through different physical setting and geometries is by no means exhaustive, hence the need to further explore its relevance in a growing technological innovation as we have today. It is believed that the outcomes of this study would find relevance in biotechnology, thermal sciences, chemical engineering, food processing, and so forth. Hence, the motivation for this research. Additionally, the utilization of the non-Newtonian fluid notion is more advantageous than Newtonian fluids, particularly for making paints, diverse polymer solutions, etc.

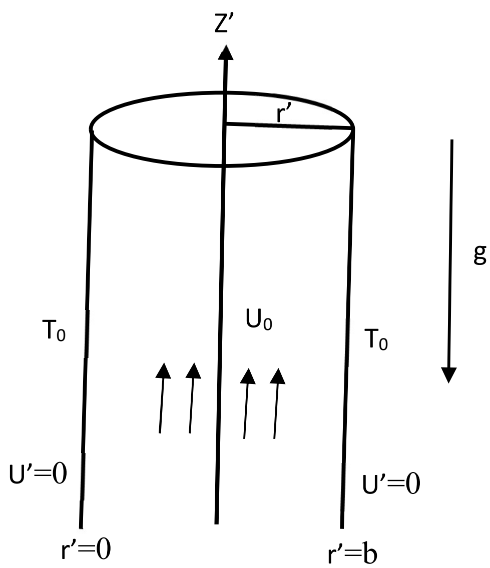

Figure 1: Schematic diagram of the flow

Problem Formulation

We consider the influence of steady state mixed convection flow of a viscous Casson fluid with exothermic chemical reaction in a vertical tube with constant pressure gradient. The z′ axis is taken along the axis of tube and r′ axis, ie radial coordinate, is taken normal to the tube as demonstrated in Figure 1. The fluid flow is assumed to be fully developed and obeys Boussinesq's approximation.

Under these assumptions, the dimensionless equations governing the analytical solution of mixed convection flow of viscous reactive fluid affected by Casson fluid impact in a vertical tube is expressed as:

$$\frac{\partial u}{\partial t} = \frac{e^{-\delta t}}{r} + \frac{GR}{\mathrm{Re}}\theta + \left(1+\frac{1}{\xi}\right)\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \quad (1)$$

$$\mathrm{Pr}\frac{\partial \theta}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right) + \lambda e^{\frac{\theta}{1+\mathrm{E}\theta}} \quad (2)$$

The boundary conditions for all the problems are:

$$\frac{\partial u}{\partial r} = 0 \quad , \frac{\partial \theta}{\partial r} = 0 \quad \text{at} \quad r = 0$$

$$U=0, \quad \theta = 0 \quad \text{at} \quad r = 1 \quad (3)$$

Methodology

The governing equations above eqns (1-3) are nonlinear and exhibit no exact solutions. Now to have the steady state form of governing equations, we have to equate $\frac{\partial u}{\partial t} = \frac{\partial \theta}{\partial t} = 0$ and as $t \to \infty$, the term $\frac{e^{-\delta t}}{r}$ disappears, so that we have:

$$\left(1+\frac{1}{\xi}\right)\frac{1}{r}\frac{d}{dr}\left[r\frac{du}{dr}\right] + \frac{Gr}{\mathrm{Re}}\theta = \Gamma \quad (4)$$

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{d\theta}{dr}\right] + \lambda e^{\frac{\theta}{1+\mathrm{E}\theta}} = 0 \quad (5)$$

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Subject to the boundary conditions

$$\frac{du}{dr}=0,\ \frac{d\theta}{dr}=0 \quad \text{at} \quad r=0$$

$$u=0, \quad \theta=0 \quad \text{at} \quad r=1 \tag{6}$$

Where Γ is the pressure gradient, Gre is the mixed convection parameter, λ is the chemical reacting parameter and ξ is the Casson fluid parameter. Also, recall $\lambda e^{\frac{\theta}{1+\mathrm{E}\theta}}$ = $\lambda\left[1+\theta+\theta^2\left(\frac{1}{2}-\mathrm{E}\right)+........\right]$

The non-dimensional quantities used are given below as follows:

$$Y=\frac{y'}{b},\theta=\frac{T-T_0}{T_1-T_0}, \quad U=\frac{u'}{U_0}, \quad \textit{where}\ U_0=\frac{\rho g\beta(T_1^{'}-T_0)b^2}{\mu}$$

$$\lambda=\frac{QC_0^{*}AEH^2}{RT_0^2}e^{(\frac{-E}{RT_0})},Gre=,\frac{g\beta(T_1^{'}-T_0)b^3}{v^2} \quad \mathrm{Re}=\frac{U_0 b}{v},\Gamma=\frac{1}{\rho}\frac{dP}{dx} \tag{7}$$

To solve equation (4) and equation (5) subject to the boundary conditions (6), we employ the use of regular perturbation method.

Now to solve the temperature equation, from equation (5) above:

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{du}{dr}\right]+\lambda e^{\frac{\theta}{1+\mathrm{E}\theta}}=0$$

By expansion, we rewrite equation (5) as:

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{d\theta}{dr}\right]+\lambda\left[1+\theta+\theta^2\left(\frac{1}{2}-\mathrm{E}\right)\right]=0 \tag{8}$$

We assume that

$$\theta=\theta_0+\lambda\theta_1+\lambda^2\theta_2 \tag{9}$$

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$$U = U_0 + \lambda U_1 + \lambda^2 U \tag{10}$$

Substituting equations (9) and (10) into equation (8), and by taking the highest power of λ to be 2, we have:

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{\partial}{\partial r}(\theta_0 + \lambda\theta_1 + \lambda^2\theta_2)\right] + \lambda + \lambda\theta_0 + \lambda^2\theta_1 + a_1(\lambda\theta_0^2 + 2\lambda^2\theta_0\theta_1) = 0 \tag{11}$$

Where $a_1 = \left(\frac{1}{2} - \mathrm{E}\right)$

Now by taking the coefficient of λ^0, λ, λ^2, we have the following:

$$\lambda^0 : \frac{1}{r}\frac{d}{dr}\left[r\frac{d\theta_0}{dr}\right] = 0 \tag{12}$$

$$\lambda : \frac{1}{r}\frac{d}{dr}\left[r\frac{d\theta_1}{dr}\right] + 1 + \theta_0 + a_1\theta_0^2 = 0 \tag{13}$$

$$\lambda^2 : \frac{1}{r}\frac{d}{dr}\left[r\frac{d\theta_2}{dr}\right] + \theta_1 + 2a_1\theta_0\theta_1 = 0 \tag{14}$$

The solution of θ_0, θ_1, θ_2 have been determined as:

$$\theta_0 = 0 \tag{15}$$

$$\theta_1 = \frac{1}{2} - \frac{r^2}{2} \tag{16}$$

$$\theta_2 = \frac{r^4 + 5}{24} - \frac{r^2}{4} \tag{17}$$

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Similarly, to solve the velocity equation from equation (4) above:

$$\left(1+\frac{1}{\xi}\right)\frac{1}{r}\frac{d}{dr}\left[r\frac{du}{dr}\right]+\frac{Gr}{Rr}\theta=\Gamma$$

We let $b_1=\left(1+\frac{1}{\xi}\right)$ and substituting the assumed solutions $\theta=\theta_0+\lambda\theta_1+\lambda^2\theta_2$ and $U=U_0+\lambda U_1+\lambda^2 U$ we have,

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{d}{dr}(U_0+\lambda U_1+\lambda^2 U_2)\right]+\frac{1}{b_1}\frac{Gr}{Rr}(\theta_0+\lambda\theta_1+\lambda^2\theta_2)=\frac{\Gamma}{b_1} \qquad (18)$$

Substituting the values of θ_0 , θ_1 and θ_2 into equation(18),we have

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{d}{dr}(U_0+\lambda U_1+\lambda^2 U_2)\right]+\frac{1}{b_1}\frac{Gr}{\text{Re}}\lambda\left(\frac{1}{2}-\frac{r^2}{2}\right)+\frac{1}{b_1}\frac{Gr}{\text{Re}}\lambda^2\left(\frac{r^4+5}{24}-\frac{r^2}{4}\right)=\frac{\Gamma}{b_1} \qquad (19)$$

Taking the coefficients of λ^0, λ, λ^2, we have

$$\lambda^0:\ \frac{1}{r}\frac{d}{dr}\left[r\frac{dU_0}{dr}\right]=\frac{\Gamma}{b_1} \qquad (20)$$

$$\lambda^0:\ \frac{1}{r}\frac{d}{dr}\left[r\frac{dU_1}{dr}\right]=-\frac{1}{b_1}\frac{Gr}{\text{Re}}\left(\frac{1}{2}-\frac{r^2}{2}\right) \qquad (21)$$

$$\lambda^2:\ \frac{1}{r}\frac{d}{dr}\left[r\frac{dU_2}{dr}\right]=-\frac{1}{b_1}\frac{Gr}{\text{Re}}\left(\frac{r^4+5}{24}-\frac{r^2}{4}\right) \qquad (22)$$

The transformed boundary conditions now become:

$\frac{dU_0}{dr}=0$ at r = 0 and $U_0=0$ at r = 1

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$\frac{dU_1}{dr} = 0$ at r = 0 and $U_1 = 0$ at r = 1

$\frac{dU_2}{dr} = 0$ at r = 0 and $U_2 = 0$ at r = 1 (23)

The solutions of U_0, U_1, U_2 have been obtained as:

$$U_0 = -\frac{\Gamma r^2}{2b_1} + D_1 r + D_2 \quad (24)$$

$$U_1 = -\frac{1}{b_1}\frac{Gr}{\mathrm{Re}}\left(\frac{r^2}{4} + \frac{r^4}{24}\right) + D_3 r + D_4 \quad (25)$$

$$U_2 = -\frac{1}{b_1}\frac{Gr}{\mathrm{Re}}\left(\frac{r^6}{720} + \frac{5r^2}{48} - \frac{r^4}{48}\right) + D_5 r + D_6 \quad (26)$$

Where the skin friction at the both plates is derived as:

$$\left.\frac{dU}{dr}\right|_{r=0} = D_1 + \lambda D_3 + \lambda^2 D_5 \quad 27)$$

$$\left.\frac{dU}{dr}\right|_{r=1} = -\frac{\Gamma}{2b_1} + D_1 + \lambda(-\frac{7Gre}{24b_1} + D_3) + \lambda^2(-\frac{61Gre}{720b_1} + D_5) \quad (28)$$

Where $D_1 = D_3 = D_5 = 0,\quad D_2 = -\frac{\Gamma}{2b_1},\quad D_4 = \frac{5Gre}{24b_1},\quad D_6 = \frac{61Gre}{720b_1}$

Recall that $\theta = \theta_0 + \lambda\theta_1 + \lambda^2\theta_2$ and $U = U_0 + \lambda U_1 + \lambda^2 U_2$

Results and Discussion

The analysis of chemically reacting fluid (exothermic fluid) on steady-state mixed convection and heat flow problem in a vertical tube is investigated in the presence of Casson fluid effects and constant pressure gradient. The variation of key parameters such as Casson fluid parameter ξ, chemical reaction parameter λ, mixed convection parameter Gre with constant pressure gradient Γ are investigated. To demonstrate the influence of

ISSN: 3043–6087. Science and Education Development Inst., Nigeria

these relevant parameters, Figures 2-5 exhibits the fluctuation of temperature and velocity distributions. The current analysis was performed by taking the default values as: λ=1, ξ=0.1, Gre=100 and Γ=1.

Figure 2 demonstrates the functions of λ on the temperature distribution. It is readily evident that when the value of λ grows, the fluid temperature rises significantly in the vertical channel. Also, as the temperature jump expands at the channel wall, the fluid surface impact diminishes, leading to an increase in the speed of fluid at the boundary of the channel. Figure 3 describe the impacts of λ on the velocity profile. It is revealed from this figure that, as the levels of chemical reaction parameter is raised, the velocity of the fluid is strengthened. This is owing to the truth that a growth in λ, which basically denotes the viscous heating parameter, corresponds to an increase in the chemical reaction's strength. Naturally speaking, these results make sense, as raising the heat source parameter indicates more heat generation from the region's surface, making the fluid to accelerate faster, thereby resulting in high performance of the system. There are many thermal applications where this result can find relevance which include power generation, space heating, cooking and industrial processes, to mention a few. Figure 4 showcases the function of Casson fluid parameter on the velocity gradient. The pattern demonstrates a decrease in the fluid flow behavior, which is true as a result of the viscous nature of the fluid. Increase in the fluid's parameter means stronger thickness of the fluid. This makes the fluid flow to be getting weaker. Casson fluids have various rheological features, which makes them more relevant to chemical, biomedical sciences, chemical engineering, pharmaceutical and so on. The action of mixed convection parameter on the velocity is portrayed in Figure 5. It is interesting to report that, increasing the mixed convection parameter makes the fluid velocity to rise first at lower plate, then reduces towards the upper plate. There is however a noticeable point of intersection at the center of the channel where the flow is independent of the mixed convection. This behavior occurs due to the asymmetric and symmetric heating at the surface of the channel plates.

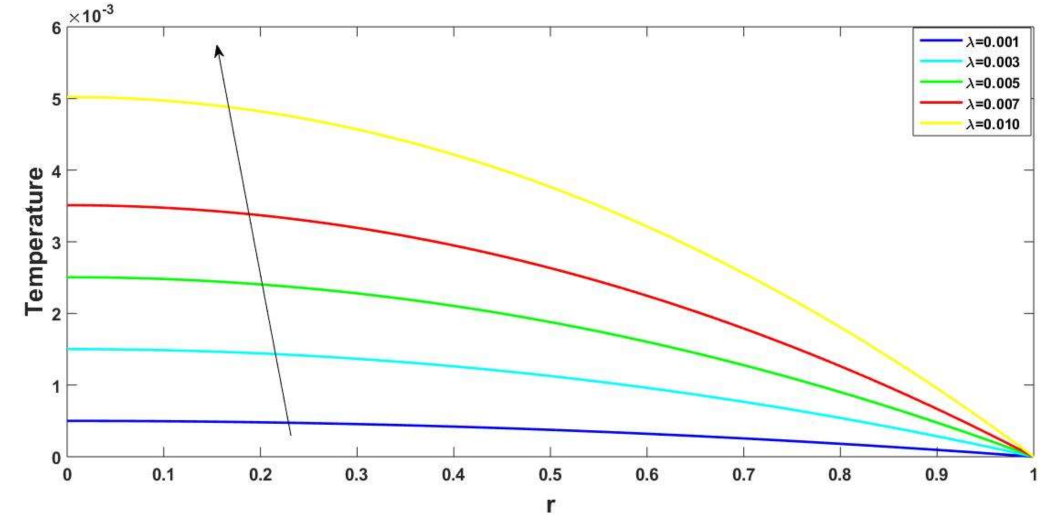

Figure 2 Influence of λ on Temperature Profile

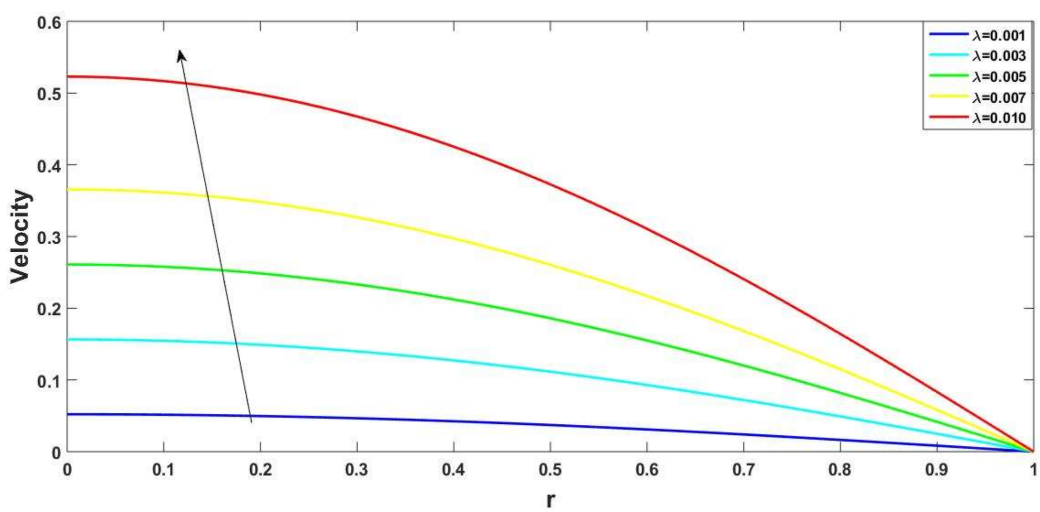

Figure 3 Influence of λ on Velocity Profile

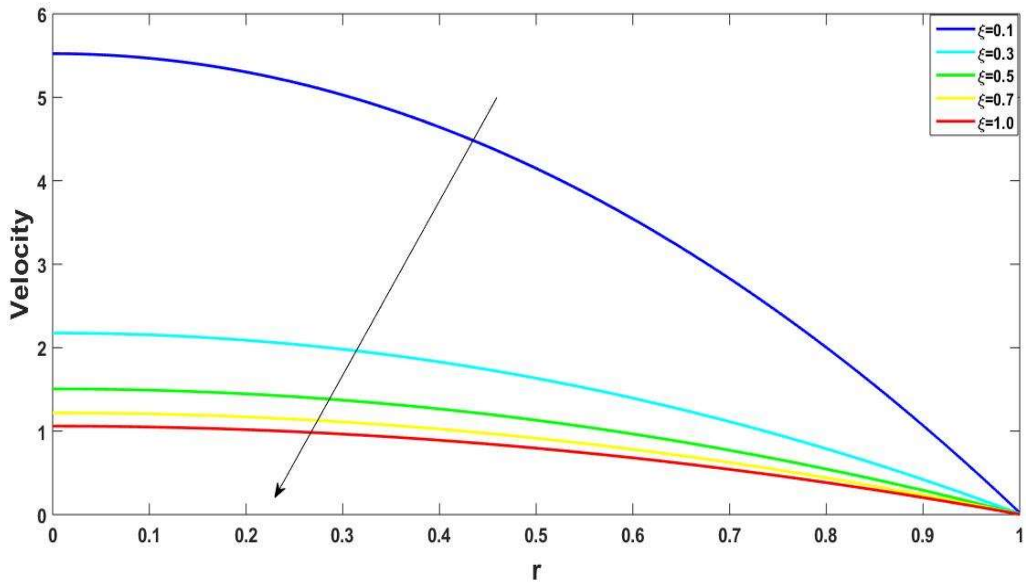

Figure 4 Influence of ξ on Velocity Profile

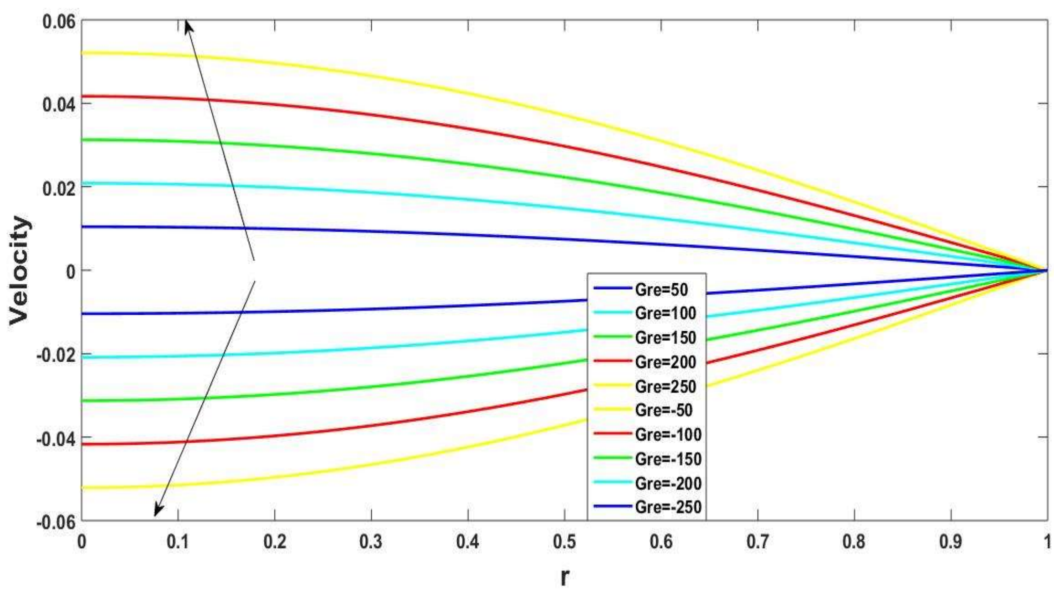

Figure 5 Influence of Gre on Velocity Profile

Conclusion

A fully developed mixed flow of a chemically reactive fluid was performed across a channel using direct integration method. The resultant nonlinear governing equations and closed form expressions for temperature and velocity have been derived. The effects of controlling parameters were displayed graphically, and the obtained results discussed thoroughly. A solid comprehension of the properties of these types of fluids is critical in the engineering and medical fields. The summary of this research is highlighted below:

i. It is concluded that an increase in the fluid temperature and velocity occurs for increasing values of chemical reacting parameter.
ii. Higher values of Casson fluid parameter is seen weaken the fluid velocity.
iii. The effect of positive mixed convection is observed to escalate the velocity of the fluid whereas increasing the negative mixed convection parameter produce a reverse trend.
iv. In the future, this study will be expanded to investigate the effects of time and heat source/sink.

References

Ahmad K. S and Jha B. K (2015). Computational Methods of Transient/steady Natural Convection Flow of Reactive Viscous Fluid in Vertical Porous Pipe. *Asian Journal of Mathematics and Computer Research.*2, 74-92.

Ahmad K., Wahid Z. and Hanouf Z. (2019). Heat transfer analysis for Casson fluid flow over stretching sheet with Newtonian and viscous dissipation, *Journal of Physics: Conference Series*, 1127(1), article 012028.

Ahmad S. K., Ojemeri G. and Hamza M. M. (2017). Mixed Convective Heat and Mass Transfer Flow of Reactive Viscous Fluid in a Vertical Porous Pipe in the Presence of Thermal Diffusion, *Asian Journal of Mathematics and Computer Research*, 17, 7387.

Ahmad S. K., Sani A. A., Muhammad I. and Usman H. (2023). Impacts of Exponentially Growing/Decaying Pressure Gradient on Mixed Convection Flow of Viscous Reactive Fluid in a Vertical Tube: A Numerical Approach, *Saudi Journal of Engineering and Technology,* 8(7), pp. 181-188.

Ajibade, A. O., Umar, A. M. (2021). Effects of viscous dissipation on a steady mixed convection Couette flow of heat generating/absorbing fluid. *Science Forum (J Pure Applied Sci)*. **21**, 67-76. http://dx.doi.org/10.5455/sf.93898

Ali A., Fatima A., Bukhari Z., Farooq H. and Abbas H. (2021). Non- Newtonian Casson pulsatile fluid flow influenced by Lorentz force in a porous channel with multiple constrictions: A numerical study, *Korea-Aust. Rheol. J*, 33,79–90.

Ali, F., Khan, I. And Sharidan S. (2014). Closed Form Solutions For Unsteady Free Convection flow Of A Second Grade Fuid over an Oscillating Vertical Plate, *PLoS ONE*, 9(2),e85099.

Amaraikannan N., Karthikeyan S. and Chaudhary D. K. (2021a). Significance of MHD radiative non-Newtonian nanofluid flow towards a porous channel: a framework of the Casson Fluid Model, *Journal of Mathematics*,1-15, Article ID 9912239.

Amaraikannan N., Karthikeyan S., Chaudhary D. K. and Kayikci S. (2021b). Analytical investigation of magnetohydrodynamic non-Newtonian type Casson nanofluid flow past a porous channel with periodic body acceleration, *Complexity*, 2021b, 1-17, Article ID 7792422.

Amjad M., Zehra I., Nadeem S. and Abbas N. (2021) Thermal analysis of Casson micropolar nanofluid flow over a permeable curved stretching surface under the stagnation region, *Journal of Thermal Analysis and Calorimetry*, 143(3), 2485–2497.

Aydin, O. and Kaya, A. (2013). MHD mixed convection of a viscous dissipating fluid about a vertical slender cylinder. *Desalination and Water Treatment,* 51(16-18), 3576-3583.

Daniel Y. S. and Daniel S. K. (2015). Effects of buoyancy and thermal radiation on MHD flow over a stretching porous sheet using homotopy analysis method, *Alexandria. Engineering Journal,* **54**, 705–712.

Das S., Banu A., and Jana R. (2021) Delineating impacts of non-uniform wall temperature and concentration on time-dependent radiation-convection of Casson fluid under magnetic field and chemical reaction, *World Journal of Engineering,* 18(5), 780–795.

Frank Kamenetskii D.A. (1969): Diffusion and Heat Transfer in Chemical Kinetics. –New York: Plenum Press.

Hamid M., Usman M., Khan Z., Haq R. and Wang W. (2019). Heat transfer and flow analysis of Casson fluid enclosed in a partially heated trapezoidal cavity, *International Communications in Heat and Mass Transfer*, 108, article 104284.

Hamza M. M., Bello S. R., Ibrahim Y., Bello A. and Tasiu R. A. (2024a). Casson fluid flow with heat sink affected by symmetric wall temperature and concentration conditions, *Journal of Engineering Technology,* (JET, 15(2), pp. 1-20.

Hamza M. M, Ojemeri G. and Ahmad S. K (2023a). Insights into an analytical simulation of a natural convection flow controlled by Arrhenius kinetics in a micro-channel, *Heliyon* 9(2023) e17628, pp. 1-13.

Hamza M. M. (2016). Free convection slip flow of an exothermic fluid in a convectively heated vertical channel. *Ain Shams Engineering J*ournal, 9(4), pp. 1413-1323. http://dx.doi.org/10.1016/j.asej.2016.08.011

ISSN: 3043–6087. Science and Education Development Inst., Nigeria

Hamza M. M, Ojemeri G. and Ahmad S. K (2023a). Insights into an analytical simulation of a natural convection flow controlled by Arrhenius kinetics in a micro-channel, *Heliyon* 9(2023) e17628, pp. 1-13.

Hamza M. M., Ahmad S. K., Ojemeri G., Usman H. and Shuaibu A. (2023b). Exploring the dynamics of an exothermic chemical reaction controlled by Arrhenius kinetics with Hall effect in a microchannel, *FUDMA Journal of Sciences,* 7(3), 112-124.

Hamza, M. M., Abdulsalam, S. and Ahmad, S. K. (2023c). Time-Dependent Magnetohydrodynamic (MHD) Flow of an Exothermic Arrhenius Fluid in a Vertical Channel with Convective Boundary Condition. *Advances in Mathematical Physics,* 2023. https://doi.org/10.1155/2023/7173925

Hamza M. M., Ojemeri G. and Shuaibu A. (2019). Mixed convection flow of viscous reactive fluids with thermal diffusion and radial magnetic field in a vertical porous annulus, *Computer Mathematics and Modelling,* **30**(3) DOI:10.1007/510598-019-09451-0

Hamza, M. M. and Shuaibu, A. (2022). Influence of chemical kinetic exponent on transient mixed convective hydromagnetic flow in vertical channel with convective boundary condition. *International Journal of Thermofluids,* 16, 100220.

Hamza, M. M., Abdulsalam, S. and Ahmad, S. K. (2023c). Time-Dependent Magnetohydrodynamic (MHD) Flow of an Exothermic Arrhenius Fluid in a Vertical Channel with Convective Boundary Condition. *Advances in Mathematical Physics*, https://doi.org/10.1155/2023/7173925

Hayat, T., Waqas, M., Shehzad, S. A., and Alsaedi, A. (2016). Mixed convection flow of viscoelastic nanofluid by a cylinder with variable thermal conductivity and heat source/sink. *International Journal of Numerical Methods for Heat & Fluid Flow,* 26(1), 214-234.

Hindebu, B., Makinde, O. D., Guta, L. Unsteady mixed convection flow of variable viscosity nanofluid in a micro-channel filled with a porous medium. *Indian J Phys,* **96**, 1749-1766 (2022)

Jafar, A. B., Shafie, S., Ullah, I., Safdar, R., Jamshed, W., Pasha, A. A., Eid, M. R. (2022). Mixed convection flow of an electrically conducting viscoelastic fluid past a vertical nonlinearly stretching sheet. *Scientific Reports,* 12(1), 14679. https://doi.org/10.1038/s41598-022-18761-0

Jamaludin A., Naganthran, K., Nazar R. and Pop I. (2020). MHD mixed convection stagnation-point flow of Cu-Al2O3/water hybrid nanofluid over a permeable stretching/shrinking surface with heat source/sink, *European. Journal Mechanical Fluids,* **84**, 71–80

Jha B. K., Ahmad K. S. and Ajibade A. O. (2011a). Transient Free-Convective Flow of Reactive Viscous Fluid in a Vertical Channel, *International Communication in Heat and Mass Transfer,* 38, 633-637

Jha B.K, Ahmad K. S, Ajibade A. O. (2011b). Transient Free Convective Flow of Reactive Viscous Fluid in a Vertical Tube. *International Communications in Heat and Mass Transfer*, 54 2880-2888.

Lavine, A. S. (1988). Analysis of fully developed opposing mixed convection between inclined parallel plates. *Wärme-und Stoffübertragung,* 23(4), 249-257.

Mandal B, Bhattacharyya, K, Banerjee, A, kumarverma A. & Gautam, A. k. (2020). MHD mixed convection on an inclined stretching plate in Darcy porous medium with soret effect and variable surface conditions, *Nonlinear Engineering,* **9**, 457-469. https://doi.org/10.1515/nleng-2020-0029

Makinde O. D. (2008) Thermal Criticality in Viscous Reactive Flows through Channels with a Sliding Wall an Exploitation of the Hermite-Pade Approximation Method. *Journal of Mathematics and Computational Model*, 47, 312–7.

Muthuraj R, and Srinivas S, (2010) Mixed Convective Heat and Mass Transfer in a Vertical Wavy Channel with Travelling Thermal Waves and Porous Medium. *Journal of Computers and Mathematics with Applications,* 59, 3516-3528.

Nandeppanavar M. M. (2018). Melting heat transfer analysis of non-Newtonian Casson fluid due to moving plate, *Eng. Computations*, 35(3), 1301-1313.

Obalalu A. M., Ajala O. A., Adeosun A. T., Aindele A. O., Oladapo O. A. and Olajide O. A. (2021). Significance of variable electrical conductivity on non-Newtonian fluid flow between two vertical plates in the coexistence of Arrhenius energy and exothermic chemical reaction, *Partial Differential Equations Applications and Mathematics*, 4, 100184, 1-9

Ojemeri G. and Abubakar J. (2024). Impacts of Casson Fluid Flow and Heat Source/Sink in an Oscillating Plate with Unequal Wall Temperature, *International Journal of Science for Global Sustainability,* 10(3), pp. 129-138. DOI: https://doi.org/10.57233/ijsgs.v10i3.709.

Ojemeri, G. and Hamza, M. M. (2022). Heat transfer analysis of Arrhenius-controlled free convective hydromagnetic flow with heat generation/absorption effect in a micro-channel, *Alexandria Engineering Journal*, **61**, pp. 12797-12811.

Ojemeri G., Omokhuale E., Hamza M. M., Onwubuya I. O. and Shuaibu A. (2023a) A Computational Analysis on Steady MHD Casson Fluid Flow Across a Vertical Porous Channel Affected by Thermal Radiation Effect, *International Journal of Science for Global Sustainability,* 9(1), https://doi.org/ 10.57233/ijsgs.v9i1.393

Ojemeri, G., Hamza, M. M., Tambuwal B. H., Bello I. and Shuaibu, A. (2023b). Influence of Soret and Radial magnetic field on natural convection of a chemically reactive fluid in an upright porous annulus, *UMYU Scientifica*, 2(3), 108-120.

Ojemeri, G., Onwubuya I. O., Shuaibu A., Omokhuale, E. and. Altine M. M. (2023c). Arrhenius-controlled heat transfer fluid provoked by porosity effect through a vertical microchannel: An analytical approach, *Continental Journal of Applied Sciences,* 18(1), 18-40.

Onwubuya I. O., Ojemeri G. and Gyegwe G. T. (2024). The role of Casson fluid flow in a magnetized oscillatory porous channel with non-uniform wall heating, *African Journal of Mathematics and Statistics Studies*, 7(3), pp. 156-167.

Saedodin, S., Biglari, M., Esfe, M. H. & Noroozi, M. J. (2013). Mixed convection heat transfer performance in a ventilated inclined cavity containing heated blocks: Effect of dispersing Al2O3 in water and aspect ratio of the block, *J. Comput. Theor. Nanosci.* 10, 2663–2675.

Sarkar S. and Endalew M. F. (2019). Effects of melting process on the hydromagnetic wedge flow of a Casson nanofluid in a porous medium, Boundary Value Problems, (1).

Sarkar S., Jana R. N. and Das S. (2020). Time-dependent entropy analysis in magnetized Cu-Al2O3/ethylene glycol hybrid nanofluid flow due to a vibrating vertical plate, *Fluid Mechanics Research*, 47(5), 419–443.

Shahzad H., Wang X., Sarris I., Iqbal K., Hafeez M. B. and Krawczuk M. (2021). Study of non-Newtonian biomagnetic blood flow in a stenosed bifurcated artery having elastic walls, *Scientific Reports*, 11(1).

ISSN: 3043–6087. Science and Education Development Inst., Nigeria