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Research Article

Development of a Hybrid Acceptance Sampling Plan Utilising the Binomial-Gumbel Type II Distribution

Aminu Bello Zoramawa, Shehu Usman Gulumbe, Nakazalle Fauziyya Nura*
Department of Statistics, Usmanu Danfodiyo University, Sokoto, Nigeria
***Corresponding Author:** nakazallefauziyya70@gmail.com

Abstract

Quality has emerged as a crucial criterion for consumers when choosing among diverse competing items and services in the marketplace. This research established a hybrid group acceptance sampling plan based on the lifetime of a product that adheres to the Binomial-Gumbel Type II distribution, with the objective of determining the probability of acceptance, minimum values, and Operating Characteristic (OC) curves. The findings indicate enhanced economic dependability in product quality testing, safeguarding producers from rejecting acceptable lots and customers from accepting inferior finished products. The mean life ratio figures also inform the producer on enhancing the quality of their product. Additionally, consumers possess greater safeguards against accepting inferior products from producers, as illustrated in the example. Comprehensive data and examples are included for illustrative purposes. A comparative analysis of the proposed Hybrid Group Acceptance Sampling Plan, employing the Binomial-Gumbel Type II Distribution, reveals a greater probability of acceptance than the current Hybrid Group Acceptance Sampling Plan based on the New Weighted Exponential Distribution. The proposed plan necessitates inspection of 16 items, while the existing plan requires 28 items, indicating that the proposed plan optimally minimizes the number of observations needed for decision-making.

Keywords: Group Acceptance Sampling Plan, Consumer's Risk, Producer's Risk, Operating Characteristic, Operating characteristic curve Mean ratio.

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Introduction

Statistical quality control plays a crucial role for the success or failure of an industry. It is extremely evident that in many cases it may not be possible to execute a hundred percent inspection. On the other hand, if nothing is tested, desired quality cannot be assured. For this end, acceptance sampling plans play a crucial role in statistical quality control. Acceptance sampling plan is a "middle path" between hundred percent inspection and no inspection at all. The acceptance sampling is concerned with accepting or rejecting a submitted lot of products on the basis of the quality of the products inspected in a sample taken from the lot. Ever since the early work of Dodge and Romig (1929), acceptance sampling has rapidly gained wide application in industry, particularly in manufacturing stages to inspect incoming materials, on-line production control and finished product quality auditing. After the pioneer work of Dodge and Romig (1929), Mood (1943) pointed out the need for more efficient inspection procedures for situations where the lot quality is not a fixed constant due to random fluctuations and quality variations in the production process. An acceptance sampling plan is a scheme that establishes the minimum values size to be used for testing. This becomes particularly important if the quality of product is defined by its lifetime (Rajagopal and Vijayadevi, 2018). Often, it is implicitly assumed when designing a sampling plan that only a single item is put in a tester. However, in practice testers accommodating a multiple number of items at a time are used because testing time and cost can be saved by testing items simultaneously. The items in a tester can be regarded as a group and the number of items in a group is called the group size. An acceptance sampling plan based on such groups of items is called a group acceptance sampling plan (GASP).

Zoramawa and Gulumbe (2021) study on Sequential Probability Sampling Plan for a Truncated Life Tests Using Rayleigh Distribution in which estimations of minimum sample, acceptance and rejection numbers are obtained, analyzed and presented to explain the usefulness of sequential plans in relation to single and double sampling plan. Probability of acceptance (Pa), Average sample number (ASN) and Average outgoing quality (AOQ) for the plans are determined.

Hybrid group acceptance sampling plan based on a truncated life test is proposed in the study of Srinivasa and Bala (2022), when the consumer's risk and other plan characteristics are given, the number of groups and acceptance number are determined. As the test termination time multiplier grows, it is seen that the minimal number of groups required lowers. Furthermore, as quality improves, the operating characteristics function grows disproportionately. When a large number of items are being examined at the same time, this Hybrid Group Acceptance Sampling Plan (HGASP) can be used. To

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maintain the quality of a product, we have to satisfy both producer and consumer. So it is desirable to protection of both the parties. The probability of rejecting a good lot is denoted by α and called producer's risk. The probability of accepting a bad lot is denoted by β and called consumer's risk. We assumed consumer's risk β is fixed to determine the number of testers (r) for acceptance sampling plan (Sandeep 2019).

Even though the acceptance sampling continually faces many challenges, among the challenges is the requirement to improve product quality while lowering production cost. In most acceptance sampling plans for a truncated life test, the major issue is to determine the sample size from a lot under consideration. If the quality characteristic is regarding the lifetime of a product, the plan becomes a life test. Traditionally, when the life test indicates that the mean life of products exceeds the specified one, the lot of product is accepted, otherwise it is rejected. It also protects the producer in the sense that lots produced at permissible level of quality have a good chance to be accepted by the plan. Acceptance sampling procedures are necessary defensive measures, instituted as protective devices against the threat of deterioration in quality (Zoramawa *et al.* 2018).

The Gumbel distribution is one of the most used models to carry out risk analysis in extreme events, in reliability tests, and in life expectancy experiments. This distribution is adequate to model natural phenomena among others. Gumbel type II distribution is a unimodal distribution with probability density function (pdf) and cumulative distribute function (cdf). The standard Gumbel distribution has been widely applied in engineering and other fields, especially in extreme-value analysis. Nadarajah and Kotz, (2004), the Beta Gumbel distribution was introduced and showed greater flexibility in explaining the variability of the tails than the Gumbel distribution. It was demonstrated that the extra parameters gave greater variability of the skewness and kurtosis.

Novelty of the research

Group acceptance sampling plans is more robust and flexible in terms of designing new acceptance sampling plans in different industrial settings for production and standardization on the production of quality products, it has a way of minimizing the cost of inspections, time and getting the minimum number more than the classical acceptance sampling plans. There is new method of improving the group acceptance sampling plan, which is called hybrid group acceptance sampling plan. Determining the minimum values that will give assurance at a specified time remains the major challenge in the area of acceptance sampling. Some authors like Srinivasa (2011), Braimah and Osanaiye (2017), Srinivasa and Bala (2022) work to determine the sample size on acceptance sampling plan. The aforementioned authors have utilized other probability distributions to proposed hybrid group acceptance sampling plans for lifetimes, this

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research work will modify the hybrid group acceptance sampling plans based on Binomial-Gumbel Type II Distribution by obtaining the probability of acceptance for the plan, determining the minimum values and using real life data and compare out with existing distribution which yield a minimum acceptance testers for the termination of the unit, operating characteristics Curve and ration for the lifetime of a product.

The aim of this research is to propose a hybrid group acceptance sampling plans (HGASP) for lifetimes based on Binomial-Gumbel type II Distribution. Balancing the interest of both the producer's risk and consumer's risk, by obtaining the probability of acceptance for Binomial-Gumbel Type II Distribution. To determine the minimum values of the proposed hybrid group acceptance sampling plans based on Binomial-Gumbel Type II Distribution. To use a real life data to assess the lot using the propose plan, to make comparison with the existing plan(s).

Methodology

Gumbel Type-II Distribution

The random variable X is said to follow a Gumbel type-II distribution with parameter α and β , where the cumulative distribution function (CDF) is given by:

$$F\left(x \mid \alpha, \beta\right) = 1 - \exp\left[-\beta x^{-\alpha}\right], x > 0, \alpha, \beta > 0, \qquad (1)$$

The corresponding probability density function (PDF) of (eq) is

$$f\left(x \mid \alpha, \beta\right) = \alpha\beta x^{-(\alpha+1)} \exp\left[-\beta x^{-\alpha}\right],\ x > 0,\ \alpha, \beta > 0. \quad (2)$$

The hazard function of Gumbel Distribution is given by:

$$\mathrm{h(x)} = \left(1/\beta\right) * \exp\left(-\left(x-\mu\right)/\beta\right) * \left(-\exp\left(-\left(x-\mu\right)/\beta\right)\right) \quad (3)$$

where
h(x) is the hazard function
μ is the location parameter
β is the scale parameter
exp() represents the exponential function

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Assume that a product's lifetime is controlled by Binomial-Gumbel Type II Distribution with the scaling parameter.

$$F(t) = \exp\left[-\left(\frac{1}{\beta}\right)\right] \qquad (4)$$

Minimum sample values

In order to obtain the minimum values n ensuring $\sigma > \sigma_0$ at the consumers' confidence level $1-\beta$, we consider the binomial distribution. The issue is to determine the smallest value of n that will satisfy the given value of $p^*\left(0 < p^* < 1\right)$ and c so that t $t_q > t_q^0$ satisfy;

$$\sum_{i=0}^{c}\binom{n}{i}p^i\left(1-p\right)^{n-i} \le 1-p^* \qquad (5)$$

where $p = F\left(t;\delta_0\right)$ is the probability of failure during the time t given $\delta_0 = t/\sigma_0$

So let take (3.4) as the time t and specified life σ so as to obtain the minimum values

$$F(t) = \exp\left[-\left(\frac{t}{\sigma_0}\right)\right] \qquad (6)$$

Assuming that the life of the product follows Binomial-Gumbel Type II Distribution with scale parameter σ. The hazard function of follows F(.) is given by:

$$F(t;\sigma) = \exp\left[-\left(\frac{\sigma}{t}\right)\right], t > 0, \sigma > 0 \qquad (7)$$

where σ is the scaled parameter.

The minimum values is determined using the following inequality

$$\sum_{i=0}^{c}\binom{n}{i}p^i\left(1-p\right)^{n-i} \le \beta \qquad (8)$$

Table 1. Presented the minimum values calculated by R Software which satisfied (3.8) inequality.

The probability of acceptance of this sampling plan is given by

$$L(P) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \quad (9)$$

In this approach, the specified values of $r = 6$, $g = 4$, c= {0,1,2,3,4,5,6,7,8,9,10}, t/σ_0= {1.0, 1.50, 2.0, 2.25, 2.50, 3.0}, P*={0.75, 0.90, 0.95, 0.99} the minimum values is determined satisfying(3.6) also data by Zoramawa *et. al.* (2018) was used to explain the results obtained.

Operating Characteristics (OC Curve)

The operating characteristics (OC) curve depicts the discriminatory power of an acceptance sampling plan. The OC curve plots the probabilities of acceptance a lot versus the fraction defective. When the OC curve is plotted, the sampling risks are obvious.

$$\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \geq 1-\alpha \quad (10)$$

Where $1-\alpha$ is a producer's confidence

Probability of Acceptance for the Binomial-Gumbel Type II Distribution

When the lifetime follows the Binomial-Gumbel type II Distribution, the probability of acceptance is given as;

$$L(P) = \sum_{i=0}^{c} \binom{n}{i} \left(e^{\left(-t/\sigma_0\right)} \right)^i \left(1 - e^{\left(-t/\sigma_0\right)} \right)^{n-i} \quad (11)$$

Proposed Hybrid Group Acceptance Sampling Plan

It's usually desirable to set up a sampling plan with both the producer's and consumer's interest in mind. The producer and consumer risks are put into consideration in order to avoid good product from being rejected or bad product being accepted. There are two samples to be considered, a first sample of size n and c as the acceptance numbers for the sample respectively. Let the number of defectives be d which is counted and compared to the sample acceptance number drawn at random from the lot (usually assumed large).

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Our proposed HGASP is described as follows: Let t_0 and σ_0 be the specified truncated time average life respectively. A lot is considered good and worthy of acceptance of the true unknown average life is more than the specified average life. Likewise, the lot is considered bad if the true unknown average life is less than the specified average life within the specified time. We assume that the lot size is large enough to sufficiently use the binomial distribution to find the probability of acceptance. In this study the consumer's risk is fixed so as not to exceed $1-p^*$, so that p* is a minimum confidence level for a consumer, the minimum values should satisfy the inequality in (3.8) which is used to complete the tables of minimum mean ratio and probability of acceptance respectively.

To calculate the probability of acceptance of our proposed sampling plan, let F be the cumulative distribution function of the lifetime of a product, then the probability that a product fails before the time t_0 denoted by p is obtained by

$$p = F(t_0) \qquad (12)$$

Now the HGASP may be denote as (n, t/σ_0). The probability of acceptance is given as:-

$$L(p) = \left[\sum_{i=0}^{c} \binom{r}{i} p^i (1-p)^{r-i} \right]^g \qquad (13)$$

The probability that the lot is accepted from Group acceptance Sampling is given by

$$P_a(p) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n_0-i} + \sum_{i=1}^{c} \binom{n_1}{x} p^x (1-p)^{n_1-x} \qquad (14)$$

Data Description

The data that was used in this research is a data adopted by Zoramawa et al (2018) using R Package Software version 4.3.2. Let consider the ordered sample of size *n*=14 from time T (in hours)

t_i: *i*=1, 2, …….14): 254, 788, 1054, 1393, 3316, 2880, 3593, 4281, 5180, 6003, 7621, 8783, 9604, 10064……..

Results and Discussion

This section present the result obtained using the method discussed above by using computations of the proposed minimum values, probability of acceptances based on the designed parameter are presented.

Table 1. Minimum value of n satisfying the inequality for the population mean using binomial distribution

		Ratio Time					
P*	C	1.0	1.50	2.0	2.25	2.50	3.0
0.75	0	3	2	2	2	2	2
	1	7	5	4	4	3	3
	2	10	7	6	5	5	5
	3	13	9	8	7	7	6
	4	16	11	9	9	8	8
	5	19	14	11	11	10	9
	6	22	16	12	12	12	11
	7	25	18	15	14	13	12
	8	28	20	17	16	15	14
	9	31	22	18	17	16	15
	10	34	24	20	19	18	15
P*	C	1.0	1.50	2.0	2.25	2.50	3.0
0.90	0	5	4	3	3	3	2
	1	9	6	5	5	5	4
	2	13	9	7	7	6	6
	3	16	11	9	9	8	8
	4	20	14	11	11	10	9
	5	23	16	13	12	12	11
	6	26	18	15	14	13	12
	7	30	21	17	16	15	14
	8	33	23	19	18	17	15
	9	36	25	21	19	18	17
	10	39	27	23	21	20	19
P*	C	1.0	1.50	2.0	2.25	2.50	3.0
0.95	0	7	5	4	3	3	3
	1	11	7	6	6	5	5
	2	15	10	8	8	7	7
	3	19	13	10	10	9	8

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	4	22	15	12	12	11	10
	5	26	18	14	14	13	12
	6	29	20	16	15	14	13
	7	32	22	18	17	16	15
	8	36	25	20	19	18	16
	9	39	27	22	21	20	18
	10	42	29	24	23	21	20
P*	C	1.0	1.50	2.0	2.25	2.50	3.0
0.99	0	10	5	5	5	5	4
	1	15	10	8	7	7	6
	2	19	13	10	9	9	8
	3	24	15	12	11	11	10
	4	28	18	15	14	14	12
	5	31	21	17	16	16	14
	6	35	23	19	17	17	15
	7	38	26	21	19	19	17
	8	42	28	23	21	21	18
	9	45	30	25	23	23	20
	10	50	33	27	25	25	22

Proposed Hybrid Group Acceptance Sampling Plan Based on Population Mean

In order to obtain the minimum values satisfying equation (5) for the same combination of p^*, t/σ_0, p as in equation (7). Let the specified average life be 700 hours and the testing time is 1000, 1500, 2000, 2250, 2500, and 3000 by selecting 10 items from the product.

Table 1: shows the minimum values of n satisfying the inequality for population mean using binomial distribution where ratios are calculated and obtained the minimum values. It shows that we can obtain a minimum of 34 product items at ratio 1.0 and 15 product items under ratio 3.0 respectively for **p* 0.75**.According to **p* 0.90**, it indicate the minimum values of 39 and 19 obtained at ratio 1.0 and ratio 3.0. Similarly, **p* 0.95** display the minimum values to be 42 product items as of ratio 1.0 and 20 product item at ratio 3.0 respectively. Regarding **p*0.99** it shows that the minimum value is 50 items and 22 items under the ratio of 1.0 and 3.0.

This shows that the consumer have more protection of accepting bad lot from producer, the lot will be accepted since its only 2 items that falls below the average life of 1000 as designed by the plan.

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The Operating Characteristics (OC Curve)
After obtaining the least number of testers r, one may be interested in determining the probability of a lot being accepted when the quality is regarded good if $\mu \geq \mu_0$ or $\frac{\mu}{\mu_0}$.
Suppose that the lifetime of a product follows the Binomial-Gumbel Type II Distribution, it is desired to design a HGASP to test if the median is greater than 1,000 hours based on a testing time of 700 hours and using 4 group. The OC values can be obtained for any sampling plan. To save space we present the OC values for sampling plans with $\frac{\mu}{\mu_0}$ = {1, 2, 3, 4}, n=8 t/s= {1.0, 1.50, 2.0, 2.25, 2.50, 3.0} and $\alpha = 0.05$ respectively.

Table 2: Operating Characteristics as Generated From R Code for Binomial-Gumbel Type II Distribution

		Ratio Time					
P*	μ / μ_0	1.0	1.50	2.0	2.25	2.50	3.0
0.75	1	0.9958	0.9462	0.7994	0.6800	0.5266	0.1096
	2	1.0000	0.9997	0.9958	0.9901	0.9805	0.9462
	3	1.0000	1.0000	0.9999	0.9997	0.9992	0.9958
	4	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997
P*	μ / μ_0	1.0	1.50	2.0	2.25	2.50	3.0
0.90	1	0.9983	0.9798	0.9288	0.8893	0.8399	0.7113
	2	1.0000	0.9999	0.9983	0.9961	0.9924	0.9798
	3	1.0000	1.0000	1.0000	0.9999	0.9997	0.9983
	4	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
P*	μ / μ_0	1.0	1.50	2.0	2.25	2.50	3.0
0.95	1	0.9992	0.9902	0.9661	0.9478	0.9251	0.8667
	2	1.0000	0.9999	0.9992	0.9981	0.9963	0.9902
	3	1.0000	1.0000	1.0000	0.9999	0.9998	0.9992
	4	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
P*	μ / μ_0	1.0	1.50	2.0	2.25	2.50	3.0
0.99	1	0.9998	0.9981	0.9935	0.9901	0.9858	0.9751
	2	1.0000	1.0000	0.9998	0.9996	0.9993	0.9981
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 2 displays the operating characteristics using generated r code for n=1:6, c=2 and t/s to be 1,2,3,4 with alpha 0.05. According to the table it shows clearly that the probability of acceptance for the lot at the ratio correspond to the producers risk, therefore we accept the entire lot.

Comparison of the Proposed Plan with other Existing Plan

In order to compare the performance of the proposed new sampling plan HGASP with the current one, i.e., Anil and Srinivasa (2022), the new HGASP requires 24 (n = rg) items; however, the existing plan requires 28 items accordingly to meet the required probability of acceptance. Also for the OC Curve, the proposed HGASP requires 16 items, whereas the existing plan requires 24 items. So the proposed sampling plan has the optimal least number of observations to make a judgment in accepting a lot than traditional HGASP as shown on tables A and B.

Table A. Comparison of sample size (n)

β	N	Existing HGASP	Proposed HGASP
0.75	G	4	4
0.90	R	7	6
0.95	C	2	2

Table B. Comparison of OC Curve

N	Existing HGASP	Proposed HGASP
G	4	4
R	6	4
C	2	2

Conclusion

On the study of hybrid group acceptance sampling plan based on the finding of this research, it can be concluded that as test termination time multiplier increases, the minimum number of testers required decreases and also the operating characteristics values increase more rapidly as the quality improves, so it was concluded that hybrid group acceptance sampling plan for lifetime based on Binomial-Gumbel Type II Distribution was accepted. This HGASP can be used when a multiple number of items at a time are adopted for a life test, and it would be beneficial in terms of test time and cost because a group of items will be tested simultaneously. Finally, on comparing the two distributions, one can find that the proposed hybrid group acceptance sampling plan based on Binomial-Gumbel type II distribution is comparatively better than the existing hybrid group acceptance sampling plan for the new weighted exponential distribution. Hence, the proposed HGASP is more cost effective and can be used conveniently.

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References

Anil, A. and Srinivasa R. B. (2022). Hybrid Group Acceptance Sampling Plan for New Weighted Exponential Distribution. *International Journal of Applied Mathematics and Statistics,* 61(2); 21-29

Braimah, O. J. and Osanaiye, P. A. (2017). Truncated Hybrid Double Acceptance Sampling Plan (THDASP) for Weibull Product Life Distribution. *American Journal of Management Science and Engineering.* 2(5), 80-88. doi: 10.11648/j.ajmse.20170205.12

Dodge, H.F., and H.G. Roming (1929). A Method of Sampling Inspection. *Bell System Technical Journal*, VIII, 613-631.

Mood, A.M. (1943). On The Dependence of Sampling Inspection Plans upon Population Distributions, Annals of Mathematical Statistics, 14, 415-425.

Nadarajah, S., and Kotz, S. (2004). The Beta Gumbel Distribution, *Mathematical Problems in Engineering,* 2004(4), 323-332, DOI: 10.1155/S1024123X04403068.

Rajagopal, S. and Vijayadevi, K. (2018). A hybrid group acceptance sampling plan for lifetimes based on transmuted exponential distribution. *International Journal of Statistics and Applied Mathematics*; 3(2): 268-273

Sandeep. K (2019). Hybrid Group Acceptance Sampling Plan on Truncated Life Tests based on Sushila Distribution. *International Journal of Current Microbiology and Applied Sciences.* 8(8), http://doi.org/10.20546/ijcmas-2019.808.04

Srinivasa R. B. and Bala S. R., (2022). A Hybrid Group Acceptance Sampling Plans For Life Tests Based On The Exponentiated Inverted Weibull Distribution. *Advances and Applications in Mathematical Sciences.* 21(5), 2545-2558

Srinivasa R. G., (2011), A hybrid group acceptance sampling plans for lifetimes based on generalized exponential distribution, *Journal of Applied Sciences*, 11: 2232-2237.

Zoramawa, A. B., Gulumbe S. U, Kantam, R. R. L., and Musa, Y (2018) Developing double acceptance sampling plans for percentiles based on inverse Rayleigh distribution. *International Journal of Statistics and Applied mathematics* 3(1):39 –44

Zoramawa, A. B., and S. U. Gulumbe (2021) On Sequential Probability Sampling Plan for a Truncated Life Tests Using Rayleigh Distribution. *Asian Journal of Probability and Statistics* 15(1): 1-7. DOI: 10.9734/AJPAS/2021/v15i130339